

## ASTRONOMY &amp; ASTROPHYSICS EXAM 2008: ADVANCED PART

This is a 2.5 hour exam, and you only need to answer 6 of the 20 questions. Each question is identified at the top with the instructor and course number so you can go directly to those questions that are relevant to the courses you took.

To avoid confusion and in keeping with previous policy, submit answers to only 6 questions. If you attempt to answer more than 6 questions, please cross out your work on the additional questions so that it is clear which ones you wish to submit.

As an additional confirmation of the six questions that you wish to submit, please check them in the Table below.

Please use only one side of each page for your answers. If you need to extend your answer to more than one page, continue your work on one of the additional pages supplied during the exam. Be sure to put your name on every page that you turn in and, if you need to use additional pages, add both the problem number and your name at the top of each page.

You may use a hand calculator on this exam.

## ANSWERS SUBMITTED

Question	Course	Six Answers Submitted <sup>(a)</sup>
1	Lin: ASTR 222	_____
2	Illingworth: ASTR 240B	_____
3	Shastry: PHYS 210	_____
4	Seiden: PHYS 216	_____
5	Laughlin: ASTR 212	_____
6	Laughlin: ASTR 204B	_____
7	Laughlin: ASTR 204A	_____
8	Laughlin: ASTR 235	_____
9	Bodenheimer: ASTR 220B	_____
10	Woosley: ASTR 220C	_____
11	Aguirre: PHYS 226	_____
12	Glatzmeier: EART 275	_____
13	Glatzmeier: PHYS 227	_____
14	Madau: ASTR 240C	_____
15	Rockosi: ASTR 260	_____
16	Mathews: ASTR 230	_____
17	Dekel: ASTR 233	_____
18	Bolte: ASTR 257	_____
19	Max: ASTR 289C	_____
20	Primack: PHYS 224	_____

<sup>(a)</sup> Please check the six questions that you are submitting.

## ASTROPHYSICS EXAM INFORMATION SHEET

### Physical constants:

speed of light in vacuum	$c$	$2.998 \times 10^8 \text{ m/s} = 2.998 \times 10^{10} \text{ cm/s}$
Gravitational constant	$G$	$6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2 = 6.67 \times 10^{-8} \text{ cm}^3/\text{g s}^2$
Elementary charge	$e$	$1.60 \times 10^{-19} \text{ C} = 4.80 \times 10^{-10} \text{ esu}$
Planck constant	$h$	$6.625 \times 10^{-34} \text{ Js} = 6.625 \times 10^{-27} \text{ erg s}$
Fine structure constant	$\alpha = e^2/\hbar c$	1/137
Boltzmann constant	$k$	$1.38 \times 10^{-23} \text{ J/K} = 1.38 \times 10^{-16} \text{ erg/K}$
Gas constant	$\mathcal{R}$	$= 8.32 \times 10^7 \text{ erg K}^{-1} \text{ mole}^{-1}$
Electron mass	$m_e$	$9.11 \times 10^{-31} \text{ kg} = 9.11 \times 10^{-28} \text{ gm}$
Proton mass	$m_p$	$1836m_e$
Electron classical radius	$r_e = e^2/m_e c^2$	$2.82 \times 10^{-15} \text{ m} = 2.82 \times 10^{-13} \text{ cm}$
Compton wavelength	$h/m_e c$	$2.426 \times 10^{-12} \text{ m} = 2.426 \times 10^{-10} \text{ cm}$
Bohr radius	$a_B = \hbar^2/m_e e^2$	$0.529 \times 10^{-10} \text{ m} = 0.529 \times 10^{-8} \text{ cm}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$5.79 \times 10^{-11} \text{ MeV/T}$
Rydberg energy	$m_e c^2 \alpha^2/2$	13.6 eV
Stephan Boltzmann const.	$\sigma_{SB} = 2\pi^5 k^4/15c^2 h^3$	$5.67 \times 10^{-8} \text{ J/s m}^2 \text{ K}^4 = 5.67 \times 10^{-5} \text{ erg/s cm}^2 \text{ K}^4 \text{ s}$
radiation constant	$a = 4\sigma_{SB}/c$	$7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$
Thompson scattering	$\sigma_T = (8\pi/3)r_e^2$	$6.65 \times 10^{-29} \text{ m}^2 = 6.65 \times 10^{-25} \text{ cm}^2$
Avogadro number	$N_A$	$6.022 \times 10^{23} \text{ mol}^{-1}$

### Astrophysical Quantities:

$M_\odot$	$2 \times 10^{33} \text{ g}$
$L_\odot$	$4 \times 10^{33} \text{ erg s}^{-1}$
$R_\odot$	$7 \times 10^{10} \text{ cm}$

### Unit conversions:

electron volt	$1.60 \times 10^{-12} \text{ erg}$
year	$3.15 \times 10^7 \text{ s}$
Joule	$10^7 \text{ erg}$
arc second	$4.848 \times 10^{-6} \text{ radians}$
Angstrom	$10^{-8} \text{ cm}$
1 AU	$1.50 \times 10^{13} \text{ cm}$
parsec	$3.08 \times 10^{18} \text{ cm}$

### Other information of questionable usefulness:

sound speed in air at 300° K	330 m/s	$3.30 \times 10^4 \text{ c/s}$
atmospheric pressure	$1. \times 10^5 \text{ N/m}^2$	
acceleration of gravity	$9.8 \text{ m/s}^2$	$980 \text{ cm/s}^2$

## Equations of interest:

Maxwell's equations	$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \quad \nabla \cdot \mathbf{H} = 0$ $\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad \nabla \cdot \mathbf{E} = 4\pi\rho$
ideal gas	$P = \rho kT / (\mu m_p) = \rho \mathcal{R}T / \mu$
blackbody	$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$
blackbody radiation density	$u = (4\sigma_{SB}/c)T^4 \equiv a_B T^4$
first law	$dQ = dE + PdV$
Schrodinger's equation	$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2\mu} \nabla^2 \Psi + U(x, y, z) \Psi$ $\left(\frac{\hbar^2}{2\mu}\right) \nabla^2 \Psi + [E - U(x, y, z)] \Psi = 0$
Friedmann's Equation	$H^2 = H_0^2 \left[ \frac{\Omega_M}{a^3} + \frac{\Omega_K}{a^2} + \frac{\Omega_R}{a^4} + \Omega_\Lambda \right]$

**Lin: ASTR 222 – PLANETARY SCIENCE (W08)**

Planets formed in protostellar disks. This question focuses on an evaluation of the physical conditions in these disks.

- (a) Derive an expression for the thickness of the disk as a function of the mid-plane temperature and the distance from the central star. Name two sources of heating which determine the disk temperature.
- (b) Use the continuity and momentum equations to show that the evolution of the disk is determined by the effective viscosity of the gas.
- (c) Discuss two challenges to the formation of a gas giant planet in a protostellar disk.

**Illingworth: ASTR 240B – GALACTIC AND EXTRAGALACTIC STELLAR SYSTEMS (W08)**

Early-type galaxies and bulges contain most of the mass in stars at the present time. Thus their buildup is of particular interest.

- (a) What is the fraction of the baryonic mass in stars in ellipticals (& bulges/spheroids) vs the fraction in disk components vs the fraction in late type galaxies, at the present time? Why is this important for thinking about galaxy formation and buildup?

Three recent results provide some challenges for understanding the processes and timescales of galaxy buildup.

First, there is a substantial fraction of the stellar mass in galaxies that is already in evolved galaxies at redshifts in the range  $z = 2$  to  $z = 3$ .

Second, there are also indications of a red sequence at redshift  $z \sim 2$ .

- (b) What are the challenges and issues regarding time scales and galaxy buildup processes that arise from these observations?

Third, recent observations show that, on average, early-type galaxies at redshift in the range  $z = 2$  to  $z = 3$  are much smaller, but denser, than those at the present time. This has resulted in much discussion but little illumination.

- (c) Why has this raised much consternation? Discuss a process (or processes) that might enable these galaxies to become today's ellipticals.

**Shastry: PHYS 210 – CLASSICAL MECHANICS**

Consider the 1-d Hamiltonian

$$H = \frac{k p^2}{2 q^2} + \frac{1}{2m} q^4.$$

Here  $k, m$  are some positive parameters. For this system

- (a) Find the Hamilton's equations of motion.
- (b) Find a canonical transformation that maps it into the Harmonic oscillator.
- (c) From this mapping, find the frequency of oscillations in terms of  $k, m$ .

**Seiden: PHYS 216 – QUANTUM MECHANICS II**

Suppose the proton's electric charge was contained in a very thin shell of radius  $r_0$ . Using first order perturbation theory, calculate the change in energy to the ground state of hydrogen due to the finite proton size. You may assume that over the region of the proton the hydrogen wave function is given by a constant

$$\Psi = \left( \frac{1}{\pi a_0^3} \right)^{1/2},$$

where  $a_0$  is the Bohr radius.

**Laughlin: ASTR 212 – DYNAMICAL ASTRONOMY (F06)**

Since its discovery by the Geneva team in 2001, HD 80606b has been a favorite example of an extreme extrasolar planet, with  $P = 111.42$  d,  $e = 0.932$ ,  $\varpi = 300.4^\circ$ ,  $i = 89^\circ$ , and  $K = 474 \text{ ms}^{-1}$ .

- (a) Assuming a solar-mass parent star, what are HD 80606b's mass and semi-major axis?
- (b) HD 80606b is thought to owe its high eccentricity to "Kozai migration" that was facilitated by the binary companion star, HD 80607 (which currently lies at a projected separation  $d \sim 1000$  AU). Explain the basic dynamical mechanism underlying a Kozai resonance.
- (c) If the system age is 5 Gyr, and if the tidal quality factor,  $Q$ , appropriate to the planet's eccentricity tide has remained constant over the system's lifetime, and if the planet's semi-major axis  $\sim 5$  Gyr ago was 3 AU (when the Kozai migration was destroyed by GR precession), provide a back-of-the-envelope estimate of the current tidal luminosity of the planet. Note that you don't need a detailed dynamical theory of tides to make this estimate.

**Laughlin: 204B – PHYSICS OF ASTROPHYSICS I**

Assume that  $\mu$  is the coefficient of shear viscosity, and

$$D_{ik} \equiv \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3}(\nabla \cdot \mathbf{u})\delta_{ik}$$

is the traceless symmetric rate of strain.

- (a) Write down a fluid momentum equation (i.e the Navier-Stokes equation) that properly incorporates the viscous force per unit mass, and briefly explain the terms in the equation.
- (b) Using the equation that you developed in part (a), write down a rough estimate of the ratio,  $r = a_{adv}/a_{visc}$  for a fluid element, where  $a_{adv}$  is acceleration arising from the advective term and  $a_{visc}$  is acceleration due to the viscous term.
- (c) Aside from numerical coefficients of order unity, the coefficient of shear viscosity,  $\mu$  is given by  $\mu \sim mv_{thermal}/\sigma$ , where  $\sigma$  is a typical collision cross section. Use this to get an estimate of the Reynold's number in a typical protoplanetary disk.

**Laughlin: ASTR 204A – PHYSICS OF ASTROPHYSICS II**

Define and explain the following terms or concepts:

- (a) The Specific Intensity,  $I_\nu$ , the Mean Intensity,  $J_\nu$ , and the Monochromatic Pressure Tensor,  $P_{ik\nu}$
- (b) The Eddington-Barbier relation
- (c) Local Thermodynamic Equilibrium
- (d) The Pulsar Dispersion Measure
- (e) The Metropolis Hastings Algorithm
- (f) The Grand Canonical Ensemble
- (g) The Electric Dipole Approximation

**Laughlin: ASTR 235 – NUMERICAL METHODS FOR ASTROPHYSICS (W07)**

Consider the following model flux-conservative equation

$$\frac{\partial u}{\partial t} = -\frac{\partial F}{\partial x},$$

with

$$F = -D\frac{\partial u}{\partial x}.$$

- (a) If  $D$  is a constant, give a qualitative drawing of the time and space evolution of  $u_0 = \exp(-(x/\sigma)^2)$ .
- (b) Write down a “forward time centered space” finite difference representation for the model equation above.
- (c) Use a von Neumann stability analysis to derive a stability criterion for your FTCS scheme, and give a physical interpretation of this criterion.

**Bodenheimer: ASTR 220B – STAR AND PLANET FORMATION (W07)**

1. A massive star with solar metal abundance starts to form in a molecular cloud core with a radius of 0.1 pc., a mass of  $100 M_{\odot}$ , a temperature of 10 K, and a mean turbulent velocity of 2 km/s.

- (a) What is the ‘thermal’ Jeans mass? What is the turbulent Jeans mass? When the cloud starts to collapse, what will be the approximate mass accretion rate onto the ‘stellar’ core, once it forms?
- (b) Once the stellar core has reached  $20 M_{\odot}$  its luminosity is  $50000 L_{\odot}$ . What is its contraction time to the main sequence in comparison to the free fall time? Take a main-sequence radius of  $5 R_{\odot}$ . What is the implication of this result?
- (c) Once the central star reaches the main sequence, what is the physical effect that may inhibit further accretion onto the star and limit its mass? Mention two possible ways of getting around this problem of forming the stars with the highest masses. If the star is of Pop. III (no metals) is there an analogous problem?

**Woosley: ASTR 220C – ADVANCED STAGES OF STELLAR EVOLUTION (S07)**

- (a) For a contracting star of uniform density (density the same throughout the star but evolving in time) supported by ideal gas pressure, determine a scaling relation between the density and central temperature. [This scaling actually holds for any polytrope of arbitrary (constant) index].
- (b) A contracting star (or protostar) of this sort (ideal gas, uniform density) will grow hotter as it grows denser. Eventually the star either reaches temperature,  $T_1$ , and ignites nuclear burning, or density,  $\rho_1$ , where degeneracy becomes important. In general, stars will hit either  $T_1$  or  $\rho_1$ , but not both at the same time. Show, however, that there is one value of the mass where  $T_1$  and  $\rho_1$  are attained at the same time. Derive this mass algebraically (for the not-so-physical case of uniform density and ideal gas pressure. Neglect degeneracy pressure, even though it is important). You need not evaluate the expression numerically, but try to give it in simple form.
- (c) From memory, what is the value of this critical main sequence mass in solar masses for *i*) hydrogen ignition, *ii*) helium ignition, *iii*) carbon ignition, and *iv*) oxygen ignition?

**Aguirre: PHYS 226 – GENERAL RELATIVITY**

The Schwarzschild metric is:

$$ds^2 = -(1 - 2M/r)dt^2 + (1 - 2M/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

- (a) Compute the christoffel symbol  $\Gamma_{tt}^r$ . (Yes, you do remember the formula. It has three derivatives of  $g_{\mu\nu}$ , one minus sign, and a  $\frac{1}{2}$ , and gives a  $\Gamma$  symmetric in the lower two indices.)

Note that the other nonvanishing christoffel symbols for the this metric are:

$$\Gamma_{rt}^t = -\Gamma_{rr}^r = \frac{M}{r^2} \left(1 - \frac{2M}{r}\right)^{-1},$$

$$\Gamma_{\theta\theta}^r = \Gamma_{\phi\phi}^r / \sin^2\theta = -r \left(1 - \frac{2M}{r}\right)$$

$$\Gamma_{\theta r}^\theta = \Gamma_{\phi r}^\phi = \Gamma_{\phi\phi}^\theta / \sin^2\theta = 1/r$$

$$\Gamma_{\theta\phi}^\phi = \cot\theta.$$

- (b) Recalling as you surely do that

$$R_{\beta\mu\nu}^\alpha = \partial_\mu \Gamma_{\beta\nu}^\alpha - \partial_\nu \Gamma_{\beta\mu}^\alpha + \Gamma_{\sigma\mu}^\alpha \Gamma_{\beta\nu}^\sigma - \Gamma_{\sigma\nu}^\alpha \Gamma_{\beta\mu}^\sigma,$$

compute  $R_{rtr}^t$  and  $R_{trtr}$ .

(Note that the other nonzero components of  $R_{\beta\mu\nu}^\alpha$  are:

$$R_{\theta t\theta}^t = R_{\phi t\phi}^t / \sin^2\theta = M/r^5,$$

$$R_{\phi\theta\phi}^\theta = 2M \sin^2\phi / r^2,$$

$$R_{\theta r\theta}^r = R_{\phi r\phi}^r / \sin^2\theta = -M/r^5,$$

and other connected to these by symmetries<sup>1</sup>. If you are doubting your result for part (a), you could use on of these to check.)

- (c) What are the components of  $R_{\mu\nu}$ ?

---

<sup>1</sup>Recall that  $R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta} = -R_{\beta\alpha\mu\nu} = R_{\alpha\beta\nu\mu}$

**Glatzmeier: EART 275 – MAGNETOHYDRODYNAMICS**

This question is about Alfvén waves.

Derive a simple linear wave equation for Alfvén waves by neglecting viscous and magnetic diffusion and assuming constant density, no buoyancy, and no rotation. Consider a uniform steady-state magnetic field,  $\mathbf{B}_0 = B_0 \hat{\mathbf{z}}$ . This field is perturbed by a small amplitude fluid velocity,  $\mathbf{v} = v(z, t) \hat{\mathbf{x}}$ . Let  $\mathbf{B}'$  and  $\mathbf{J}'$  be the resulting perturbations in the field and current density, respectively.

- (a) Write the linear MHD induction equation for  $\mathbf{B}'$ . (Recall that  $\eta = 0$ .)
- (b) Take the curl of the expression in (a) to get an equation for the rate of change of  $\mathbf{J}'$  in terms of the vorticity,  $\boldsymbol{\omega} = \nabla \times \mathbf{v}$ , assuming the magnetic permeability,  $\mu$ , is constant.
- (c) Write the linear momentum equation, including the Lorentz force. (Recall that  $\nu = 0$ .)
- (d) Take the curl of the expression in (c) to get an equation for the rate of change of  $\boldsymbol{\omega}$  in terms of  $\mathbf{J}'$ .
- (e) Using the expressions in (b) and (d), write a wave equation for  $\boldsymbol{\omega}$  and another for  $\mathbf{J}'$ , in terms of the Alfvén velocity.

**Glatzmeier: PHYS 227 – FLUID DYNAMICS**

Consider an open cylinder of radius  $r_0$  and height  $z_0$  filled with a constant density ( $\rho_0$ ) liquid that is in “solid body” rotation. The axis of rotation is aligned with the axis of the cylinder and oriented parallel to a constant gravitational acceleration,  $g$ .

- (a) Write the three components of the momentum conservation equation in cylindrical coordinates ( $r$ ,  $\phi$ ,  $z$ ), where  $r$  is the cylindrical radius relative to the axis,  $\phi$  is the angular coordinate and  $z$  is the height above the bottom of the cylinder.
- (b) Solve for the pressure within the liquid as a function of  $r$  and  $z$  and in terms of the atmospheric pressure ( $p_0$ , the pressure on the free surface of the liquid), the dimensions of the cylinder ( $r_0$ ,  $z_0$ ) and the angular velocity of the cylinder,  $\Omega$ .

**Madau: ASTR 240C – GALACTIC & EXTRAGALACTIC STELLAR SYSTEMS**

The cosmological Jeans mass  $M_J$  (the total mass of baryons and dark matter contained within a sphere of diameter equal to the Jeans wavelength) is

$$M_J = \frac{\pi}{6\sqrt{\rho_M}} \left( \frac{\pi k_B T_e}{\mu m_p G} \right)^{3/2},$$

where  $T_e(t)$  is the temperature of the intergalactic medium,  $k_B$  the Boltzmann constant,  $\mu$  the mean molecular weight for primordial gas,  $m_p$  the proton mass, and  $\rho_M(t)$  the proper matter density.

- (a) What is  $M_J$  just after recombination and how does it scale with time?
- (b) If intergalactic gas begins cooling adiabatically at redshift 150, how does the Jeans mass scale with redshift at later epochs?
- (c) What happens at reionization?

**Rockosi: ASTR 260 – INSTRUMENTATION FOR ASTRONOMY**

The Keck 10 m telescope and the Lick 3 m telescope both have focal ratio approximately  $f/15$ .

- (a) If you have a CCD camera that has a square CCD with 4096 pixels on each side, what is the field of view of the camera on the 10 m and on the 3 m?
- (b) You are given money to put a new CCD in your camera. You shop around and have two options: one CCD with pixels that are  $50\ \mu\text{m}$  and another with  $15\ \mu\text{m}$  pixels. The CCD with  $15\ \mu\text{m}$  pixels is twice the price of the one with  $30\ \mu\text{m}$  pixels, but both are the same size. You are told that you must be able to use this camera on both the 10 m and the 3 m, and that you must be able to take advantage of conditions when the seeing FWHM at Keck is 0.2 arcseconds. Which CCD do you choose, and why?
- (c) You decide to build a spectrograph to get information about the near-ultraviolet spectra of galaxies. Your target population of galaxies are approximately evenly spread over the sky, and they emit very little flux in the ultraviolet. You have two choices for your CCD detector. Option number 1 has 2048 pixels in each dimension and a readnoise of 4 electrons. Option number 2 has 4096 pixels in each dimension but a readnoise of 8 electrons. Which do you choose, and why?

**Mathews: ASTR 230 – Low Density Astrophysics (S07)**

Assume that a bright O star of constant ionizing photon luminosity  $\mathcal{L}$  ( $\text{s}^{-1}$ ) suddenly turns on at time  $t = 0$  and begins to radiate into an essentially infinite cloud of neutral hydrogen with uniform number density  $n$  that remains fixed as the gas is ionized.

- (a) Derive a differential equation that describes the velocity  $dR/dt$  of the ionization front as it propagates spherically away from the star. The propagation of the front depends on the ionizing flux at the front, which is attenuated by recombinations that occur within the already ionized sphere. To correct for this effect, assume that the luminosity of ionizing photons incident on the ionization front is  $\mathcal{L} - \alpha_B n^2 (4/3)\pi R^3$ , where  $\alpha_B \text{ cm}^3 \text{ s}^{-1}$  is the Case B recombination coefficient.
- (b) Solve the differential equation for  $R(t)$ . Does  $R$  approach the Stromgren radius  $R_s$  as  $t \rightarrow \infty$ ? If not, why not?

**Dekel: ASTR 233 – Physical Cosmology (W07)**

Assume an EdS cosmology, and a power-law power spectrum of initial density fluctuations,  $P_k \propto k^n$ .

- (a) What is the growth rate of the fluctuations  $D(z)$ ?
- (b) What is the mass dependence of the rms fluctuation  $\sigma_0(M)$  at a given redshift?
- (c) Use (a) and (b) to estimate of the redshift dependence of the typical halo mass collapsing at that redshift,  $M_*(z)$ . What is it for  $n \simeq -2$ , valid on galactic scales?

**Bolte: ASTR 257 – MODERN OBSERVATIONAL TECHNIQUES**

- (a) What is the expression for the signal-to-noise ratio for a source measured in an aperture with radius  $r$  in a CCD image?
- (b) What is the difference in V-band magnitude for two stars which have a ratio of intensities in the V-band of 50?

An  $R = 23$  star observed with the Prime Focus Camera at the 3m produces 80 photoelectrons per second in the  $R$  band at zenith. The  $R$ -band sky brightness at Lick is  $20.3 \text{ mag/arcsec}^2$ . The PFCam pixel scale is  $0.30 \text{ arcseconds/pixel}$ , the readout noise is  $5e^-$  and the inverse gain of the system is  $2.5 e^-/\text{DN}$ .

- (i) What is the rate of detected  $e^-/\text{pixel}$  from the sky in the  $R$  band?
- (ii) What are the values of the three main components of noise for the measurement of an  $R = 23$  star measured in an aperture with radius  $r = 6 \text{ pixels}$ ?

**Max: ASTR289C – Adaptive Optics**

This problem looks at sky coverage and anisoplanatism.

A key quantity determining the broad utility of an AO system for general astronomical use is the so-called “sky coverage fraction.” The isoplanatic angle,  $\theta_0$  is an important parameter determining the sky coverage fraction.

- (a) Define the isoplanatic angle, and give an expression for it. Don’t worry about the numerical constants out front, but be sure to include all the relevant physical parameters.
- (b) Describe why the isoplanatic angle is important in determining the sky coverage fraction. Write down a notional equation relating the isoplanatic angle and the sky coverage fraction (Make sure to define all of your terms.)
- (c) Draw a sketch showing the physical origin of anisoplanatism. Show why high-altitude turbulence has more effect on anisoplanatism than low-altitude turbulence does.
- (d) How does the mean square wavefront error  $\sigma^2$  increase as the angle  $\theta$  between the target and the guide star increases? What is the power-law scaling typical of?
- (e) What is a typical size of the isoplanatic angle? (Specify what wavelength you are considering.) What types of limitations does the isoplanatic angle place on the operation of astronomical AO systems at different wavelengths? Give both conceptual and quantitative answers.
- (f) Explain why the sky coverage fraction increases when laser guide stars are used. Illustrate with a modification to your equation in part b) and with a figure.

**Primack: PHYS 224 – PHYSICAL COSMOLOGY**

This problem looks at the relativistic Doppler shift.

Use Lorentz transformations to derive the redshift and blueshift.

- (a) Consider a source at rest with respect to the observer, i.e. with four-velocity  $u^\alpha = (1, \vec{0})$ . Now consider a moving source, i.e. one in a Lorentz boosted frame. What is the source four-velocity?
- ((b) Energy is a scalar, so the energy of a particle measured by an observer is the projected four momentum  $E = -\mathbf{p} \cdot \mathbf{u} = -p^\alpha u_\alpha$ , where  $p^\alpha$  is the particle four-momentum in some frame and  $u^\alpha$  is the four-velocity (boost) of that frame with respect to the observer. Check this for a frame at rest with respect to the observer by calculating it for a massive particle 1) at rest, 2) with velocity  $\vec{v}$ . The results should look familiar.
- (c) Now allow the frame to move with velocity  $\vec{v}$  and consider a photon of energy  $E_e$  emitted from a source at rest in that frame. Relate  $E_e$  in the boosted source frame to the energy  $E_o$  the observer measures. As  $v \rightarrow 0$  you should find the usual Doppler shift formula. Define a new parameter  $z = E_e/E_o - 1$ . Is  $z$  positive or negative for a source receding (radially) from the observer? approaching (radially) the observer? At what angle between the source velocity and the line of sight from the observer does  $z$  change from negative to positive? In the Galilean case this angle is  $90^\circ$  – there is no transverse Doppler shift. In the Lorentz transverse case what is the shift to lowest order in  $v$ ?

ADVANCED ASTROPHYSICS EXAM ADDITIONAL PAGE 2008

PROBLEM NUMBER: \_\_\_\_\_ YOUR NAME: \_\_\_\_\_ 2007

ADVANCED ASTROPHYSICS EXAM ADDITIONAL PAGE 2008

PROBLEM NUMBER: \_\_\_\_\_ YOUR NAME: \_\_\_\_\_ 2007

**ADVANCED ASTROPHYSICS EXAM ADDITIONAL PAGE 2008**

PROBLEM NUMBER: \_\_\_\_\_ YOUR NAME: \_\_\_\_\_ 2007