YOUR NAME: ______________________________ 2006

ASTRONOMY & ASTROPHYSICS EXAM 2006: ADVANCED PART

This is a 3 hour exam, and you only need to answer 6 of the 17 questions. Each question is identified at the top with the instructor and course number so you can go directly to those questions that are relevant to the courses you took.

To avoid confusion and in keeping with previous policy, submit answers to only 6 questions. If you attempt to answer more than 6 questions, please cross out your work on the additional questions so that it is clear which ones you wish to submit.

As an additional confirmation of the six questions that you wish to submit, please check them in the Table below.

Please use only one side of each page for your answers. If you need to extend your answer to more than one page, continue your work on one of the additional pages supplied during the exam. Be sure to put your name on every page that you turn in and, if you need to use additional pages, add both the problem number and your name at the top of each page.

You may use a hand calculator on this exam.

ANSWERS SUBMITTED

Question	Course	Six Answers Submitted[a]
1	224	____
2	240C	____
3	226	____
4	E 275	____
5	220B	____
6	230	____
7	220C	____
8	240B	____
9	207	____
10	289	____
11	257	____
12	212	____
13	235	____
14	204A	____
15	204B	____
16	222	____
17	260	____

[a] Please check the six questions that you are submitting.

ASTROPHYSICS EXAM INFORMATION SHEET

Physical constants:

speed of light in vacuum	c	2.998×10^8 m/s $= 2.998 \times 10^{10}$ cm/s
Gravitational constant	G	6.67×10^{-11} m^3/kg s^2 $= 6.67 \times 10^{-8}$ cm^3/g s^2
Elementary charge	e	1.60×10^{-19} C $= 4.80 \times 10^{-10}$ esu
Planck constant	h	6.625×10^{-34} Js $= 6.625 \times 10^{-27}$ erg s
Fine structure constant	$\alpha = e^2/\hbar c$	$1/137$
Boltzmann constant	k	1.38×10^{-23} J/K $= 1.38 \times 10^{-16}$ erg/K
Gas constant	$\mathcal{R}$	$= 8.32 \times 10^7$ erg K^{-1} mole^{-1}
Electron mass	m_e	9.11×10^{-31} kg $= 9.11 \times 10^{-28}$ gm
Proton mass	m_p	$1836 m_e$
Electron classical radius	$r_e = e^2/m_e c^2$	2.82×10^{-15} m $= 2.82 \times 10^{-13}$ cm
Compton wavelength	$h/m_e c$	2.426×10^{-12}m $= 2.426 \times 10^{-10}$cm
Bohr radius	$a_B = \hbar^2/m_e e^2$	0.529×10^{-10} m $= 0.529 \times 10^{-8}$ cm
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.79×10^{-11} MeV/T
Rydberg energy	$m_e c^2 \alpha^2/2$	13.6 eV
Stephan Boltzmann const.	$\sigma_{SB} = 2\pi^5 k^4/15c^2h^3$	5.67×10^{-8} J/s m^2 K^4 $= 5.67 \times 10^{-5}$ erg/s cm^2 K^4 s
radiation constant	$a = 4\sigma_{SB}/c$	7.56×10^{-15} erg cm^{-3} K^{-4}
Thompson scattering	$\sigma_T = (8\pi/3) r_e^2$	6.65×10^{-29} m^2 $= 6.65 \times 10^{-25}$ cm^2
Avogadro number	N_A	6.022×10^{23} mol^{-1}

Astrophysical Quantities:

$M_\odot$	2×10^{33} g
$L_\odot$	4×10^{33} erg s^{-1}
$R_\odot$	7×10^{10} cm

Unit conversions:

electron volt	1.60×10^{-12} erg
year	3.15×10^7 s
Joule	10^7 erg
arc second	4.848×10^{-6} radians
Angstrom	10^{-8} cm
1 AU	1.50×10^{13} cm
parsec	3.08×10^{18} cm

Other information of questionable usefulness:

sound speed in air at 300 K	330 m/s	3.30×10^4 c/s
atmospheric pressure	$1. \times 10^5$ N/m^2	
acceleration of gravity	9.8 m/s^2	980 cm/s^2

Equations of interest:

Maxwell's equations	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \cdot \mathbf{B} = 0$
	$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \qquad \nabla \cdot \mathbf{D} = 4\pi\rho$
ideal gas	$P = \rho kT/(\mu m_p) = \rho \mathcal{R} T/\mu$
blackbody	$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}$
blackbody radiation density	$u = (4\sigma_{SB}/c)T^4 \equiv a_B T^4$
first law	$dQ = dE + PdV$
Schrodinger's equation	$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2\mu}\nabla\Psi + U(x,y,z)\Psi$
	$(\frac{\hbar^2}{2\mu})\nabla\Psi + [E - U(x,y,z)]\Psi = 0$
Friedmann's Equation	$H^2 = H_0^2\left[\frac{\Omega_M}{a^3} + \frac{\Omega_K}{a^2} + \frac{\Omega_R}{a^4} + \Omega_\Lambda\right]$
useful Lorentz invariants	P, dn/ν, $u^\mu u_\mu$, $\partial_\mu A^\mu = \frac{1}{c}\frac{\partial \phi}{\partial t} + \nabla \cdot \mathbf{A}$
	$F^{\mu\nu}F_{\mu\nu} = 2(B^2 - E^2)$

1. **Joel Primack: ASTR 224 – Origin and Evolution of the Universe**

   (a) Suppose that the universe is known to have an age of 14 billion years, and assume for simplicity that it has a critical density of matter and no cosmological constant. What is the distance today to the horizon?

   (b) The cosmic background radiation's last scattering surface was at redshift 1000. At that time, what was the distance from the last scattering surface to the matter that will become our galaxy?

   (c) Now suppose that you were to do these same calculations for a universe with the same Hubble constant but with $\Omega_{\text{matter}} = 0.3$ and $\Omega_\Lambda = 0.7$. Without actually plugging in numbers, say whether the answers to (a) and (b) would be greater or less.

2. **Piero Madau: ASTR 240C – Galactic and Extragalactic Stellar Systems**

The Kompaneets equation,

$$\frac{\partial \mathcal{N}}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}\left[x^4\left(\frac{\partial \mathcal{N}}{\partial x} + \mathcal{N}^2 + \mathcal{N}\right)\right],$$

governs the effect of Compton scattering on a photon energy distribution. Here, $\mathcal{N}$ is the photon occupation number, $x = h\nu/kT_e$, ν is the photon frequency, T_e is the gas temperature, and y is the Compton parameter. If the radiation spectrum is a blackbody with temperature $T \ll T_e$, show that:

(a) the terms $\mathcal{N}$ and $\mathcal{N}^2$ can be neglected compared to $\partial\mathcal{N}/\partial x$;

(b) in the Rayleigh-Jeans limit of the Planck spectrum, the Sunyaev-Zeldovich effect corresponds to a decrease in temperature, $\Delta T/T = -2y$;

(c) for a galaxy cluster with Thomson optical depth $\tau = 0.01$ and temperature $kT_e = 7.6$ keV, point (b) above implies $\Delta T = -0.08$ mK.

3. **Anthony Aguirre: ASTR 226 – General Relativity**

Real objects do not have negative mass, and large quantities of negative mass would flagrantly violate other aspects of physics (such as causality, and the 2nd law of thermodynamics). Nonetheless, we can write down the metric of a point negative mass:

$$ds^2 = -(1 + 2\mu/r)dt^2 + (1 + 2\mu/r)^{-1}dr^2 + r^2 d\Omega^2,$$

where $\mu \geq 0$. This is just the usual Schwarzschild metric with $M = -\mu$.

(a) Consider Goldmund hovering at rest at $r = 3\mu$ and fixed θ, ϕ. Suppose he sends signals, once per second by his watch, to Narcissus, who is at rest at $r \gg 2\mu$ and fixed θ, ϕ. How many seconds pass according to Narcissus between the signals?

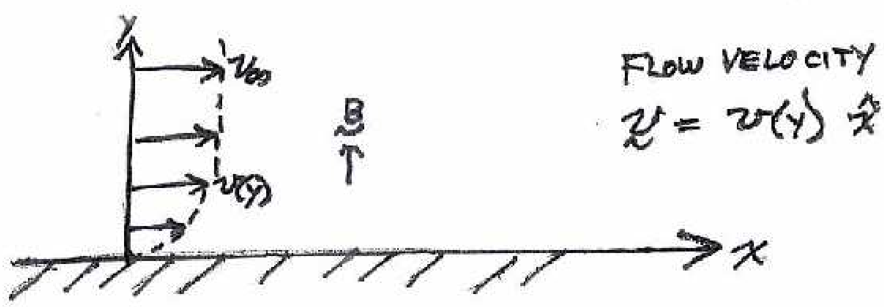
(b) Make a sketch, in $r - t$ coordinates, of the light cones both inside and outside $r = 2\mu$. Is the object a black hole, i.e. is there any region at $r > 0$ that cannot be seen by Narcissus?

(c) Without doing an explicit calculation, present an argument as to whether stable circular orbits exist near this object.

4. **Gary Glatzmaier: Earth 275 – Magnetohydrodynamics**

Hartman Boundary Layer

Consider a steady-state shear-layer in a conducting fluid adjacent to a wall:



The flow velocity is in the x-direction with $\mathbf{v} = v(y)\hat{\mathbf{x}}$. Far from the wall (large y) $v \longrightarrow v_\infty =$ constant. At the wall ($y = 0$), the velocity $v = 0$. There is an imposed, constant, uniform magnetic field $\mathbf{B} = B\hat{\mathbf{y}}$. The electrical conductivity (σ), density (ρ), and viscosity (ν) are constants. There is no electric field, and pressure is only a function of x.

(a) Give the mathematical expression for the current density ($\mathbf{J}$) and the Lorentz force for this problem.

(b) Give the x-component of the momentum equation.

(c) Give (b) in the limit of large y.

(d) Define $\delta \equiv (\rho\nu/\sigma B^2)^{1/2}$, which is a measure of the boundary layer depth, and define $U \equiv v - v_\infty$. Rewrite the equation in (b) in terms of U and δ.

5. **Peter Bodenheimer: Astron 240B Star and Planet Formation**

   (a) How do we know observationally that disks exist around a large number of young stars? Give at least 3 observational tests. How does one determine observationally what the range of disk lifetimes is? What is the implication of the deduced lifetimes with regard to giant planet formation?

   (b) Consider a nearly Keplerian disk in cylindrical coordinates (R, Z). The gravity arises from the central star only, which has mass M. Write the equation for hydrostatic equilibrium in the vertical (Z) direction, at a point (R, Z) where the density is ρ and the ideal gas pressure is P.

   (c) A disk is generally assumed to evolve by viscous processes. Name one physical mechanism that can cause a disk to evolve. Give a formula for the estimated theoretical disk evolution time. Put in numbers for a typical disk and compare your disk lifetime with observations.

6. **Jason X. Prochaska: Astron 230 Low Density Astrophysics**

Consider an OB star in the Milky Way which lies far behind a neutral Hydrogen gas cloud with density $n_H = 10\,\mathrm{cm}^{-3}$.

(a) Using the equation of radiative transfer, derive an expression for the Intensity of the star I_ν at Earth. Express your answer in terms of the optical depth τ_ν (where $d\tau_\nu = -\kappa_\nu ds$) and the Intensity at the star $I_\nu(0)$.

(b) The equivalent width of a transition with central wavelength λ_0 is defined to be

$$W_\lambda = \frac{\lambda_0^2}{c} \int\limits_0^\infty \left[1 - \frac{I_\nu}{I_\nu(0)}\right] d\nu \tag{1}$$

Estimate $W_\lambda(\mathrm{Ly}\alpha)$ in mÅ for an optically thin cloud with size $\ell = 10^{10}\mathrm{cm}$. You should know that the opacity from Lyα is:

$$\kappa_\nu = \sigma_\nu n_{HI} = \frac{\pi e^2}{m_e c} f_{Ly\alpha} \phi_\nu n_{\mathrm{HI}} \tag{2}$$

where $f_{Ly\alpha} = 0.416$, $\lambda_{Ly\alpha} = \lambda_0 = 1215.67$Å, and ϕ_ν is the line profile.

7. **Stan Woosley: Astron 220C Nucleosynthesis**

A typical classical nova is a white dwarf star of about 1 solar mass and radius 6 x 10^8 cm accreting at 10^{-9} solar masses per year. The thickness of the accreted layer is so small compared with this radius so that one may take the acceleration due to gravity and the radius to be constant. A nuclear runaway will occur when the product of temperature and density at the base of the accreted layer reaches a pressure of 10^{19} dyne cm^{-2}. What is the critical mass required for runaway and how often will the nova repeat? Give a simple scaling argument that will show how the interval between outbursts would increase or decrease as the mass of the white dwarf increased (leave the radius constant).

8. **Garth Illingworth: Astron 240B Galactic and Extragalactic Stellar Systems**

Archaeology vs direct observation

We can learn about galaxy buildup by age-dating stars in our Galaxy as well as in nearby galaxies (how do we do this? and in which galaxies?). Unlike archaeologists and paleoanthropologists, we can also observe galaxy buildup directly. Contrast and discuss the advantages and limitations of the two approaches. What are the timescales explored by the two approaches? What physical insights are gained from the two approaches? Which approach is likely to most help our understanding of galaxy formation and evolution and why?

9. **Garth Illingworth: Astronomy 207 Future Directions**

Decadal Survey

Approximately every 10 years, the National Academy conducts an "Astronomy Decadal Survey" through the National Research Council. Give brief answers to the following questions.

(a) What is the Decadal Survey?

(b) Who funds it?

(c) Who is involved?

(d) Why is it so important?

(e) What is its defining feature?

(f) What was recommended in the 2000 Decadal Survey?

(g) What will be done this Decade?

(h) Why was the 2000 Decadal Survey less successful?

(i) What would you think should be done differently in 2010?

(j) What would you like to see recommended in the next Decadal Survey?

10. **Claire Max: Astron 289 Adaptive Optics**

Anisoplanatism

(a) Define the isoplanatic angle, and give an approximate expression for it. How does the isoplanatic angle scale with observing wavelength?

(b) Draw a rough sketch of the physical origin of anisoplanatism, and describe it in words.

(c) Write an equation for the contribution of anisoplanatism to the wavefront error.

(d) How does a laser guide star help to overcome the effects of anisoplanatism? Are there astronomical science programs for which laser guide stars are not useful? What are they? Why?

11. **Mike Bolte: Astron 257 Modern Observational Techniques**

A $R = 20$ star observed with ESI (the Keck imaging spectrograph) produces 2300 detected photo-electrons per second in the broadband R filter at the zenith. The R-band sky brightness at Mauna Kea is 20.9 mag/arcsec^2. The pixel scale is 0.15 arcseconds/pixel, the readout noise is 4e$^-$ and the inverse gain of the system is 1.3 e$^-$/DN.

(a) What is the rate of detected e$^-$/pixel from the sky in the R band?

(b) What is the rate of detected e$^-$ from an $R = 26$ magnitude star, neglecting extinction by the atmosphere?

(c) Using the symbols defined below in Table 1, write the expression for signal-to-noise ratio in the brightness measurement of a point-source with a CCD in an aperture r.

(d) Assume that you are measuring all of the light for the $R = 26$ magnitude star in an aperture with a radius of 4 pixels. At what exposure time does the measurement become 'sky dominated'?

(e) What is the expression for the exposure time required to reach a given S/N in terms of the variables in the table below?

(f) How does the exposure time to a given S/N scale with the measurement aperture radius (for the sky-limited case)?

R_*	count rate from star	e$^-$/second
R_{sky}	count rate from background	e$^-$/second/pixel
t	exposure time	seconds
r	radius of aperture	pixels
G	inverse-gain	e$^-$/DN
D	dark current	e$^-$/pixel/sec
RN	Readout noise	e$^-$pixel

12. **Greg Laughlin: Astron 212 Astrophysical Dynamics**

A 1 $M_{\oplus}$ planet with an $a = 1\,\mathrm{AU}$ orbit has an orbital inclination $i = 45°$. The central star has a mass of $1 M_{\odot}$.

(a) Assume that $\varpi = 90°$ when the planet crosses the line of sight to Earth. Sketch the geometry of the situation and sketch a plot of the radial velocity curve of the star.

(b) If the orbit is circular, what is the half-amplitude, K (in m/s) of the stellar reflex velocity?

(c) If the orbit has $e = 0.5$, what are the maximum and minimum values of the stellar reflex velocity?

13. **Greg Laughlin: Astron 235 Numerical Methods**

Consider the following ordinary differential equation (ODE):

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 \,,$$

subject to initial conditions: $y(0) = 1$; $dy(0)/dx = 2$; $d^2y(0)/dx^2 = 4$.

(a) Re-express the above equation as a set of coupled first-order ODEs.

(b) Construct a second-order finite difference scheme for solving the system of ODEs.

(c) Use this scheme to integrate the system from $x = 0$ to $x = 0.2$ in a single step.

14. **Greg Laughlin: Astron 204A Physics of Astrophysics I: Radiation**

A particular species of atom has energy levels (for the neutral state) with statistical weight factors $g_N = 2N^2$, and $E_N = (1 - N^{-2})I_0$.

(a) For gas at the photosphere of a star (which is in local thermodynamic equilibrium throughout, and which has a grey atmosphere) we observe that the number density ratio for states $N = 2$ compared to $N = 1$ is $n_2/n_1 = 2$. What is the ratio n_2/n_1 at optical depth $\tau = 2$?

(b) As the excitation level becomes large, what happens to the ratio n_m/n_1?

(c) Explain the meaning to your answer to (b). What (qualitatively) is the resolution to the dilemma that has arisen?

15. **Greg Laughlin: Astron 204B Physics of Astrophysics II: Gas Dynamics**

Briefly explain (giving quantitative criteria where appropriate):

(a) Convective instability

(b) Parker instability

(c) Magnetorotational instability

(d) Rayleigh-Taylor instability

(e) Kelvin-Helmholtz instability

(f) Toomre Q

(g) Rotational Instability

16. **Doug Lin: Astron 222 Planetary Science**

Jupiter's eccentricity modulates on a time scale less than 1 Myr. How do its longitude of periapse and semi major axis vary? Briefly describe the cause of this modulation. Under what condition would this physical process lead to a much larger amplitude eccentricity modulation? Give one example in the solar system for such enhanced modulations.

17. **Connie Rockosi: Astron 260 Instruments for Astronomy**

The expression for the resolving power R of a spectrograph, $\frac{\delta\lambda}{\lambda}$, is $\frac{d_{col} m\lambda}{cos(\alpha)\phi D_t \sigma}$ where m is the diffraction order in which the grating is used, d_{col} is the diameter of the collimator mirror, $cos(\alpha)$ is the angle of incidence of the light at the grating, ϕ is the angular width of the spectrograph slit, D_t is the diameter of the telescope, and σ is the spacing of the ruling of the grating (in mm/line, etc.)

The Shane 3m has a focal ratio f/17, and the median diameter of a seeing-limited point-source image is 2". You want to design a spectrograph that can achieve a resolving power of at least 1500 at 5000Å . To save money, you have to design your spectrograph around one of two gratings found in the back of a drawer on Mt. Hamilton. Both have 250 lines/mm. Grating A has dimensions 230mm x 210mm, grating B has dimensions 306mm x 408mm. Both are blazed at 5000Å in first order, and space constraints at the 3m cassegrain focus force you to use them at $\alpha = 30°$.

(a) Describe how you would set the free parameters in the design (d_{col}, m, ϕ) to achieve your goal for the resolving power for spectrograph designs based on both gratings. What sets d_{col}? Discuss the pros and cons of your choices, and suggest ways that relaxing the restrictions given above would improve your spectrograph.

(b) How would the performance of your spectrograph change if you transport it, unchanged, from the 3m to Keck? (Keck has diameter 10m and nominal seeing 0.7". You can ignore the difference between Keck's f/15 focal ratio and the 3m's f/17 focal ratio.) How would you change it to be better-suited for Keck? Briefly describe how difficult or easy those changes would be.

(c) Back at the 3m, you need to convert your single-object spectrograph to a multi-object spectrograph. Describe, qualitatively, how your design would need to change.

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