

## PRIMACK

1(a). In such an EdS model, the present epoch horizon is  $2c/H_0 = 3ct_0$  to the distance to the horizon is 3(14 billion) light years = 42 billion light years.

(b) The distance to the cosmic background radiation sphere would be only slightly less than 42 billion light years today, so when the universe was 1001 times smaller this distance was about 42 million light years.

(c) The distance to the horizon is the integral from 0 to  $t_0$  of  $dt/a$  which equals the integral from 0 to 1 of  $da/(a^2H)$ . Since  $H$  is always smaller for the flat  $\Lambda=0$  case compared to the EdS case of  $\Lambda=0$ , the answers to (a) and (b) would be greater. (The answers actually are about 66 billion light years for the same  $H_0 = 47$  km/s/Mpc and 66 million light years.)

2. Standard textbook material, but I'm assuming this is a closed book exam. Equate mean free time  $t_f = 1/(n\sigma v)$  to the Hubble time  $= (G\rho)^{-1/2} = M_{Pl}/T^2$  where the Planck mass  $M_{Pl} = 10^{19}$  GeV and noting that the weak cross section  $\sigma = G_F^2 T^2$  and the Fermi constant  $G_F = 10^{-5} GeV^{-2}$ . This gives  $T=1$  MeV, and the corresponding time is about 1 second.

MADAU-

Maria has solution

AGUIRRE-

Maria has solutions

GLATZMAIER-

Maria has solutions

BODENHEIMER—

a) 1) direct resolved image in optical from HST, 2) infrared excess in spectrum of young star 3) Keplerian velocity profile observed from line observations in CO in radio 4) UV excess in young stars caused by accretion of disk material onto star. The range of disk lifetimes is determined by observing the fraction of stars in young clusters of various ages (determined by stellar evolutionary tracks) that have near or mid infrared excess. The median lifetime is about 3 Myr which implies that giant planet formation times must be 5 Myr or less.

b)

$$\frac{\partial P}{\partial Z} = -\frac{GM\rho Z}{(R^2 + Z^2)^{3/2}}$$

c) Magnetorotational instability (if field coupled to gas).  $t_{\text{evol}} \approx R^2/\nu$  where  $R$  is the radial scale and  $\nu$  is the viscosity. If the viscosity is approximated by the alpha-formula,  $\nu = \alpha c_s H$  then for a typical disk at 5 AU with  $\alpha = 10^{-3}$   $t_{\text{evol}} \approx 5 \times 10^5$  yr with a temperature of 150 K and  $H/R = 0.07$ .

ANS:a) The semimajor axes are 16 and 360 AU and the specific angular momenta are  $2.27 \times 10^{20}$  and  $1.14 \times 10^{21}$ . The orbital velocity is about 2 km/s. The system could have formed in a cluster environment by fragmentation during collapse of a molecular cloud to produce the inner binary, and then capture of the low-mass companion.

b) 0.58 AU (using the mass-luminosity relation to estimate the luminosity of an 0.8 solar mass star and finding the distance where the flux equals the solar constant). This orbit is almost certainly stable in a 16 AU binary. Around the low-mass companion the orbit in the habitable zone would be tidally locked and circular, and again almost certainly stable.

PROCHASKA (Q+A)

- **Jason Prochaska: Astron 230 Low Density Astrophysics**

Consider an OB star in the Milky Way which lies far behind a neutral Hydrogen gas cloud with density  $n_H = 10 \text{ cm}^{-3}$  and temperature  $T = 8000\text{K}$ .

(a) Using the equation of radiative transfer, derive an expression for the Intensity of the star  $I_\nu$  at Earth. Express your answer in terms of the optical depth  $\tau_\nu$  (where  $d\tau_\nu = -\kappa_\nu ds$ ) and the Intensity at the star  $I_\nu(0)$ .

**Solution:**

The radiative transfer equation is:

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$

Substituting for  $ds$  and integrating, we have:

$$I_\nu = I_\nu(0)e^{-\tau_{\nu r}} + \int_0^{\tau_{\nu r}} \frac{j_\nu}{\kappa_\nu} e^{-\tau_\nu} d\tau_\nu$$

where  $\tau_{\nu r}$  is the total optical depth through the cloud.

(b) The equivalent width of a transition with central wavelength  $\lambda_0$  is defined to be

$$W_\lambda = \frac{\lambda_0^2}{c} \int_0^\infty \left[ 1 - \frac{I_\nu}{I_\nu(0)} \right] d\nu$$

Estimate  $W_\lambda(\text{Ly}\alpha)$  in  $\text{m}\text{\AA}$  for a cloud with size  $\ell = 10^{10}$  cm. You should know that the opacity from Ly $\alpha$  is:

$$\kappa_\nu = \sigma_\nu n_{\text{HI}} = \frac{\pi e^2}{m_e c} f_{\text{Ly}\alpha} \phi_\nu n_{\text{HI}}$$

where  $f_{\text{Ly}\alpha} = 0.416$ ,  $\lambda_{\text{Ly}\alpha} = \lambda_0 = 1215.67\text{\AA}$ , and  $\phi_\nu$  is the line profile.

**Solution:** At  $T = 8000\text{K}$ , the cloud is predominantly neutral so that the column density of HI gas is  $N_{\text{HI}} = \ell n_{\text{H}} = 10^{11} \text{cm}^{-2}$ . The student should realize (or calculate) that this column density is small enough that we are in the optically thin limit. For example, the peak optical depth of the line profile is

$$\tau_0 = 0.015 N_{\text{HI}} \lambda f / b = 6 \times 10^{-3}$$

where  $b = \sqrt{(2kT/m)}$ .

In this case,

$$W_\lambda = \frac{\lambda_0^2}{c} \int_0^\infty \tau_{\nu r} d\nu = \frac{\lambda_0^2}{c} \int_0^\infty \kappa_{\nu r} \ell d\nu$$

The integral integrates trivially over the line profile  $\phi_\nu$ , and we have:

$$W_\lambda = \frac{\pi e^2}{m_e c^2} (f \lambda^2)_{\text{Ly}\alpha} n_{\text{HI}} \ell$$

Plugging and chugging, I get  $0.54 \text{ m}\text{\AA}$ .

ILLINGWORTH – no answers supplied for 207 or 240B

WOOSLEY – no answers supplied for 220C

MAX – answers supplied in pdf format only.

BOLTE – no answers

LAUGHLIN – no answers to three but answer to 204A:

(a) Using the Eddington approximation ( $q(\tau) \equiv \frac{2}{3}$ ), we have that  $T^4 = \frac{3}{4}T_{eff}^4(\tau + \frac{2}{3})$ . Hence at  $\tau = 2$ ,  $T(2) = 2^{1/4}T_{eff}$ . With our LTE assumption, the Boltzmann excitation equation gives  $n_2/n_1 = (g_2/g_1) \exp^{\Delta E/kT}$ . At  $T_{eff}$ ,  $n_2/n_1 = 2$ . Hence,  $\Delta E = -\ln(0.5)kT_{eff}$ .

Therefore, at  $\tau = 2$ ,  $n_2/n_1 = 4 \exp^{\ln(0.5)/2^{1/4}} = 2.23$ .

(b) As the excitation level becomes large,  $n_m/n_1 \rightarrow \infty$ .

(c) The divergence of the partition function reflects the large phase space available to atoms in highly excited states, which dominates the finite Boltzmann factor. The resolution to this dilemma is to realize that high values for  $N$  correspond to very large atoms, and in a stellar gas, the approximation of isolated atoms fails for large  $N$ . The problem is solved by following Debye and assuming that the only levels that are bound are those contained within the average volume available to the ions.

LIN – answers for 2 of three questions

Answer 1: The longitude of periape precesses and its rate of precession is modulated on the same time scale as that of eccentricity modulation. The amplitude of semi major axis modulation is much smaller. The cause for this modulation is due Jupiter's secular interaction with Saturn. Due to this interaction, the orbits of both planets are not closed. A net amount of angular momentum is exchanged between them. The direction of magnitude of angular momentum exchange is determined by the relative longitude of the two planets' peri apse. Because the change is small, each synodic encounter between the two planets is correlated with its preceding encounters. If the periods of Jupiter and Saturn are commensurate (ie their ratio is that between two small integers), they are in mean motion resonance and the amplitude of both eccentricity and semi major axis modulations will be much larger on a much shorter time scale. The orbital configuration between Io and Europa is an excellent example of a 2:1 mean motion resonance.

answer 2 The red color on the surface of Mars is caused by the oxidation of its crust. In this hypothesis, Mars once had an extensive atmosphere with considerable amount of water molecules. The water molecules became photo dissociated with the hydrogen escaping the Mars' gravity field. The residual oxygen combined with iron and silicates to provide the redish substances on the surface of Mars. Recent missions provided strong evidences that liquid water once flown on the surface of Mars. In order for water to attain a liquid state, the Marsian atmosphere must have been at least a large fraction of the Earth atmosphere. In comparison with the atmospheric content of Mars today, a substantial atmospheric loss may be inferred. The main difference between the Earth and Mars is the much larger magnetic field and gravity of the former. The particles trapped in the Earth magnetosphere provides a shield against the dissociating photons. The much large gravity also prolong the escape rate for the free hydrogen

such that they are more likely to recombine with the oxygen before escaping. The Moon has much smaller gravity than Mars. Even the free oxygen would escape so they cannot effectively oxydize the lunar surface.

## ROCKOSI

a) For these restrictions, the max collimator diameter is set by the small (perpendicular to the ruling direction) dimension of the grating. Grating B gives  $R=1500$  for a 2" slit in first order, with  $d_{col}$  306mm. For grating A, it is necessary to narrow the slit, and the tradeoff is losing light. Also possible to use the grating in second order, but the efficiency goes down, as the blaze peak in second order is at 2500A. On the other hand, grating A results in a larger spectrometer, making it more likely to have flexure problems, and requiring larger and therefore more expensive optics. More lines/mm in a grating the size of grating B would give the desired resolving power in the smaller-sized spectrograph. Going to higher order would also help, though there is an efficiency penalty.

b)  $D_t$  goes from 3m to 10m, so  $R$  goes down. The easy thing to do is make the slit more narrow, which helps get back  $R$ . More lines/mm in the grating is possible; the required ruling is 833 1/mm, well within the available ruling density of reflection gratings. Holographic ruled gratings allow even more possibility.

c) The beam size ( $d_{col}$ , and possible the width of the grating) need to be increased to make room for the multiple spectra. Making a pupil at the grating would reduce the spectrograph size. Making a fiber, rather than slitmask, spectrograph would also reduce the required beam size.

Rockosi question 2 answer

a) bandgap is too small, IR photon doesn't have enough energy elevate an electron into the bandgap.

b) the small bandgap of HeCdTe gives more dark current than Si with its larger bandgap.

c) Thermal emission is an increasingly dominant source of background at wavelengths near or greater than 2 microns, so tuning the cutoff so that the detector is sensitive to only the desired IR photons is a better idea than leaving it sensitive to 20 microns. Spitzer's thermal background is much lower because it is outside the atmosphere.

d) how far off-axis the source is (or, for extended sources, its angular size) and the the wavelength window,  $\lambda/\Delta\lambda$ . So interferometry is easier at longer wavelengths, and for smaller wavelength windows, but the penalty is the reduction in S/N for smaller wavelength windows.