

## ASTRONOMY &amp; ASTROPHYSICS EXAM 2007: ADVANCED PART

This is a 2.5 hour exam, and you only need to answer 6 of the 16 questions. Each question is identified at the top with the instructor and course number so you can go directly to those questions that are relevant to the courses you took.

To avoid confusion and in keeping with previous policy, submit answers to only 6 questions. If you attempt to answer more than 6 questions, please cross out your work on the additional questions so that it is clear which ones you wish to submit.

As an additional confirmation of the six questions that you wish to submit, please check them in the Table below.

Please use only one side of each page for your answers. If you need to extend your answer to more than one page, continue your work on one of the additional pages supplied during the exam. Be sure to put your name on every page that you turn in and, if you need to use additional pages, add both the problem number and your name at the top of each page.

You may use a hand calculator on this exam.

## ANSWERS SUBMITTED

Question	Course	Six Answers Submitted <sup>(a)</sup>
1	222	_____
2	240B	_____
3	212	_____
4	204B	_____
5	204A	_____
6	235	_____
7	220B	_____
8	220C	_____
9	P226	_____
10	240C	_____
11	260	_____
12	230	_____
13	233	_____
14	289C	_____
15	P224	_____
16	257	_____

<sup>(a)</sup> Please check the six questions that you are submitting.

**Physical constants:**

speed of light in vacuum	$c$	$2.998 \times 10^8 \text{ m/s} = 2.998 \times 10^{10} \text{ cm/s}$
Gravitational constant	$G$	$6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2 = 6.67 \times 10^{-8} \text{ cm}^3/\text{g s}^2$
Elementary charge	$e$	$1.60 \times 10^{-19} \text{ C} = 4.80 \times 10^{-10} \text{ esu}$
Planck constant	$h$	$6.625 \times 10^{-34} \text{ Js} = 6.625 \times 10^{-27} \text{ erg s}$
Fine structure constant	$\alpha = e^2/\hbar c$	1/137
Boltzmann constant	$k$	$1.38 \times 10^{-23} \text{ J/K} = 1.38 \times 10^{-16} \text{ erg/K}$
Gas constant	$\mathcal{R}$	$= 8.32 \times 10^7 \text{ erg K}^{-1} \text{ mole}^{-1}$
Electron mass	$m_e$	$9.11 \times 10^{-31} \text{ kg} = 9.11 \times 10^{-28} \text{ gm}$
Proton mass	$m_p$	$1836m_e$
Electron classical radius	$r_e = e^2/m_e c^2$	$2.82 \times 10^{-15} \text{ m} = 2.82 \times 10^{-13} \text{ cm}$
Compton wavelength	$h/m_e c$	$2.426 \times 10^{-12} \text{ m} = 2.426 \times 10^{-10} \text{ cm}$
Bohr radius	$a_B = \hbar^2/m_e e^2$	$0.529 \times 10^{-10} \text{ m} = 0.529 \times 10^{-8} \text{ cm}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$5.79 \times 10^{-11} \text{ MeV/T}$
Rydberg energy	$m_e c^2 \alpha^2/2$	13.6 eV
Stephan Boltzmann const.	$\sigma_{SB} = 2\pi^5 k^4/15c^2 h^3$	$5.67 \times 10^{-8} \text{ J/s m}^2 \text{ K}^4 = 5.67 \times 10^{-5} \text{ erg/s cm}^2 \text{ K}^4 \text{ s}$
radiation constant	$a = 4\sigma_{SB}/c$	$7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$
Thompson scattering	$\sigma_T = (8\pi/3)r_e^2$	$6.65 \times 10^{-29} \text{ m}^2 = 6.65 \times 10^{-25} \text{ cm}^2$
Avogadro number	$N_A$	$6.022 \times 10^{23} \text{ mol}^{-1}$

**Astrophysical Quantities:**

$M_\odot$	$2 \times 10^{33} \text{ g}$
$L_\odot$	$4 \times 10^{33} \text{ erg s}^{-1}$
$R_\odot$	$7 \times 10^{10} \text{ cm}$

**Unit conversions:**

electron volt	$1.60 \times 10^{-12} \text{ erg}$
year	$3.15 \times 10^7 \text{ s}$
Joule	$10^7 \text{ erg}$
arc second	$4.848 \times 10^{-6} \text{ radians}$
Angstrom	$10^{-8} \text{ cm}$
1 AU	$1.50 \times 10^{13} \text{ cm}$
parsec	$3.08 \times 10^{18} \text{ cm}$

**Other information of questionable usefulness:**

sound speed in air at 300° K	330 m/s	$3.30 \times 10^4 \text{ c/s}$
atmospheric pressure	$1. \times 10^5 \text{ N/m}^2$	
acceleration of gravity	9.8 m/s <sup>2</sup>	980 cm/s <sup>2</sup>

## Equations of interest:

Maxwell's equations  $\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \quad \nabla \cdot \mathbf{H} = 0$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad \nabla \cdot \mathbf{E} = 4\pi\rho$$

ideal gas  $P = \rho kT / (\mu m_p) = \rho \mathcal{R}T / \mu$

blackbody  $B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$

blackbody radiation density  $u = (4\sigma_{SB}/c)T^4 \equiv a_B T^4$

first law  $dQ = dE + PdV$

Schrodinger's equation  $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2\mu} \nabla^2 \Psi + U(x, y, z) \Psi$

$$\left(\frac{\hbar^2}{2\mu}\right) \nabla^2 \Psi + [E - U(x, y, z)] \Psi = 0$$

Friedmann's Equation  $H^2 = H_0^2 \left[ \frac{\Omega_M}{a^3} + \frac{\Omega_K}{a^2} + \frac{\Omega_R}{a^4} + \Omega_\Lambda \right]$

**Doug Lin: ASTR 222 – Planetary Science**

In a so-called “minimum mass solar nebula” (MMSN) hypothesis, the surface density of the gas and dust are  $\Sigma_g = 2000(r/1 \text{ AU})^{-3/2} \text{ g cm}^{-2}$  and  $\Sigma = 10\eta(r/1 \text{ AU})^{-3/2} \text{ g cm}^{-2}$ .

- (a) What effect are we accounting for if we adopt the dimensionless factor  $\eta = 1$  for disk radii  $r < 3.5 \text{ AU}$  and  $\eta = 4$  for  $r > 3.5 \text{ AU}$ ?
- (b) The temperature distribution in the MMSN is  $T = 270(r/1 \text{ AU})^{-1/2} \text{ K}$ . Why do grains in this environment have a tendency to drift radially toward the host star?
- (c) Quantitatively determine the drift time scale of 1-meter size silicate grains (assume a density  $\rho_g = 3 \text{ g cm}^{-3}$ ).

**Illingworth: ASTR 240B – Galactic and Extragalactic Stellar Systems**

This is, broadly, a question about galaxy formation and evolution.

- (a) Where are most of the baryons in the current universe?
- (b) Of the baryons in galaxies now, most are in ellipticals and bulges. What fraction is in these components? Where are the remainder? What does this tell you already about galaxy formation and evolution?
- (c) When did the baryons get assembled into galaxies? This is actually complicated, so provide times/redshifts for the different types of galaxies.
- (d) At what epoch (age and redshift) do we see the first substantial evidence for a large population of galaxies? What is their connection to today's population of early-type galaxies? What distinguishes the epoch around  $z \sim 2 - 3$  from earlier times and now?
- (e) What are some of the challenges for the current theories and semi-analytic models of galaxy formation and evolution? What do you think is the most important observational program to undertake that would help us understand how galaxies evolved to be what they are today?

**Laughlin: ASTR 212 – Dynamical Astronomy**

A  $1 M_{\text{Jup}}$  gaseous planet is in orbit around a  $1M_{\odot}$  black hole. The orbit has a period of 5 days and an eccentricity,  $e = 0.5$ .

- (a) If we neglect the spin of the planet and the star, what is the orbital angular momentum of the system? How is this angular momentum affected during the process of tidal circularization?
- (b) Again, neglecting planetary and stellar spin, what is the energy (kinetic plus potential) of the star-planet system? How is this energy affected during the process of tidal circularization?
- (c) The planet is observed to have a luminosity of  $10^{28}$  ergs. Assuming that this is all due to ongoing tidal dissipation, estimate the timescale for circularization.

**Laughlin: ASTR 204B – Physics of Astrophysics I (Gas Dynamics)**

Imagine an infinite uniform cloud of H<sub>2</sub> gas with number density  $n = 10^4 \text{ cm}^{-3}$ . The temperature is 200K.

- (a) By arguing that gravitational collapse will occur when the pressure readjustment time scale exceeds the free-fall timescale, find the smallest mass perturbation that is subject to gravitational collapse.
- (b) A linearized analysis of the hydrodynamical governing equations for this problem shows that the dispersion relation is

$$\omega^2 = k^2 c_s^2 - 4\pi G \rho_0 .$$

Explain the terms in this equation, and explain how it is related to the result that you derived above.

- (c) Imagine that the cloud is also threaded by a magnetic field, and that the conditions of ideal MHD hold. At what minimum characteristic field strength,  $B_0$ , do we expect the result of part (a) to be significantly altered?

**Laughlin: ASTR 204A – Physics of Astrophysics II (Radiation)**

Define and explain the following terms or concepts:

- (a) The specific intensity,  $I_\nu$ .
- (b) The Transfer Equation.
- (c) Local Thermodynamic Equilibrium.
- (d) Kirchoff's law.
- (e) Kinetic Temperature.
- (f) The Partition Function.

**Laughlin: ASTR 235 – Numerical Methods for Astrophysics**

Consider the partial differential equation:

$$\frac{\partial u}{\partial t} = -v \frac{\partial u}{\partial x}.$$

- (a) What is the general analytic form for the solution to this equation, and how can this solution be interpreted physically?
- (b) Write down a “forward-time centered-space” finite difference representation for the partial differential equation above.
- (c) Use a von Neumann stability analysis to assess whether your forward-time centered-space scheme is numerically stable.

**Bodenheimer: ASTR 220B – Star and Planet Formation**

The presence of dust is an important effect in star formation.

- (a) Assume a star forms with a normal component of interstellar dust, about 1% by mass. At each of the following stages, explain the main physical process by which the dust influences the structure and evolution: i) the transparent phase of protostar collapse, ii) the accretion phase of protostar collapse, iii) the equilibrium disk around a young star.
- (b) Where is the interstellar dust produced?
- (c) In the early Universe the dust was not present. How would evolution during the three phases mentioned above be changed?

**Woosley: ASTR 220C – Advanced Stages of Stellar Evolution**

The element samarium ( $Z = 62$ ) is not near a closed shell and has 7 stable isotopes. The 30 keV neutron capture cross section and solar abundance (with respect to  $10^6$  Si atoms) of each are:  $^{144}\text{Sm}$  (92 mb, 0.00742),  $^{147}\text{Sm}$  (1000 mb, 0.0371);  $^{148}\text{Sm}$  (278 mb, 0.0270);  $^{149}\text{Sm}$  (1454 mb, 0.0332);  $^{150}\text{Sm}$  (419 mb, 0.0179);  $^{152}\text{Sm}$  (378 mb, 0.0641); and  $^{154}\text{Sm}$  (293 mb, 0.0545). An examination of the periodic chart shows that  $^{144}\text{Sm}$  is a p-isotope,  $^{148,150}\text{Sm}$  are s-only,  $^{154}\text{Sm}$  is r-only; and  $^{147,149,152}\text{Sm}$  are r,s.

- (a) Why, in terms of a nuclear physics model, are the cross sections of  $^{147,149}\text{Sm}$  much larger than those for  $^{148,150}\text{Sm}$ ?
- (b) What percentage of  $^{149}\text{Sm}$  is due to the s-process and what percentage is due to the r-process?
- (c) Where in nature - what kind of star, in what shell, under what circumstances - might the s-process portion of these isotopes be produced?
- (d) What is/are the nuclear reaction(s) that provide(s) the neutrons there?

**Aguirre: PHYS 226 – General Relativity**

Consider the line element

$$ds^2 = -f^2(\theta)dt^2 + R^2(t)(d\theta^2 + \sin^2\theta d\phi^2) = g_{\mu\nu}dx^\mu dx^\nu,$$

for some functions  $f(r)$  and  $R(t)$ .

- (a) What are the components of the 4-velocity  $u^\mu$  of an observer at rest at  $\theta = \phi = 0$ ?
- (b) Calculate the Christoffel symbols  $\Gamma_{\alpha\alpha}^0$  (i.e.  $\Gamma_{00}^0$ ,  $\Gamma_{11}^0$ , and  $\Gamma_{22}^0$ ) ‘directly’ using the standard equation:

$$\Gamma_{\mu\nu}^\alpha = \frac{1}{2}g^{\alpha\beta} (\partial_\mu g_{\beta\nu} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}).$$

You can include  $f$  and  $R$ , and their derivatives.

- (c) Derive the 0-component of the geodesic equation by varying the action  $S = \int d\tau$  (and using the Euler-Lagrange equations if you like). Read off the  $\Gamma_{\alpha\alpha}^0$ , and check that they agree with your direct calculation.

**Madau: ASTR 240C – Galactic & Extragalactic Stellar Systems**

- (a) We appear to be living in a flat Universe with present-day matter density  $\Omega_m(0) = 0.24$  and vacuum density  $\Omega_\Lambda(0) = 0.76$ . In this Universe, at what redshift were the two contributions equal, i.e.  $\Omega_m(z) = \Omega_\Lambda(z)$ ? What are the values of  $\Omega_m(z)$  and  $\Omega_\Lambda(z)$  at redshift 10?
- (b) Radiation propagating in a fully ionized medium for a time interval  $dt$  is scattered according to  $d\tau = -n_e\sigma_T c dt$ , where  $\tau$  is the Thomson optical depth and the other symbols take their usual meanings ( $\sigma_T = 6.65 \times 10^{-25} \text{ cm}^2$ ). Calculate the optical depth to redshift 10 assuming a baryonic density parameter  $\Omega_b = 0.04$ , a Hubble constant  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , and neglecting the cosmological constant.

**Rockosi: ASTR 260 – Instrumentation for Astronomy**

You would like to understand the spectral energy distribution of some astrophysical source over as broad a range in wavelength as possible. Naturally this source, whatever it is, holds the key to all unsolved problems in astrophysics, so getting all the necessary data for your investigation is of the utmost importance. You should specify a specific class of object (galaxies, GRBs, white dwarfs, etc.) if it would help formulate your answers to the following questions:

- (a) Outline at least one observational program to obtain the data you need. You may use multiple facilities, and do not have to obtain the data all at once. Include a justification for your choice of facility, instrument or technique for each part of your program.
- (b) Provide an assessment of the relative difficulty (or perhaps even impossibility) of obtaining the different observations required. Alternatively, you may provide an assessment of the relative difficulty of your chosen observational program vs. other options you considered for obtaining the same or equivalent data.
- (c) Discuss three technical issues that have to be overcome in order to get your observations. Possible issues include, but are not limited to, the atmosphere and detector technology. The solutions may (and are likely to be) already in common use for astronomical observations. Just describe the issues, what the solutions are, and why each works.

**Mathews: ASTR 230 – Low Density Astrophysics**

Consider a simple ion in an HII region that has only two bound states with a forbidden line transition of energy  $E_2 - E_1 = h\nu_{12}$ . Let  $A_{21}$  be the rate of spontaneous radiative decay and  $q_{21}$  be the deexcitation rate coefficient for electron collisions.

- (a) What is the critical density  $n_c$  for this transition? Derive an expression for the line emissivity  $j_{12}$  (ergs/cm<sup>3</sup> sec ster) valid for all densities.
- (b) Derive and physically interpret the asymptotic forms of this expression in the low ( $n \ll n_c$ ) and high ( $n \gg n_c$ ) density limits. How does  $j_{21}$  depend on  $n_e$  in these two limits? [Ignore stimulated emission and entries into the levels by recombinations.]

**Dekel: ASTR 233 – Physical Cosmology**

- (a) A dark-matter halo has a mass  $M$  contained in a virial radius  $R$ . Use the virial theorem to derive its virial velocity  $V$ .
- (b) Assuming a spherical collapse, what is the mean mass density within the virial radius in terms of the mean mass density in the universe?
- (c) Based on (a) and (b), derive the virial relations between  $M$ ,  $V$  and  $R$ .
- (d) How do the virial relations depend on redshift?
- (e) Assume that the halos are isothermal, and that both the mass-to-light ratio and the spin parameter  $\lambda$  are independent of mass. Obtain the scaling relations between luminosity, circular velocity and radius for disk galaxies:  $L \propto V_{circ}^3 \propto R_{disk}^3$ .

**Max: ASTR 289C – Adaptive Optics and Its Application**

Consider the following remark: “Even the largest ground-based astronomical telescopes have no better resolution than an amateur telescope you might use in your back yard!”

- (a) Describe the physics behind this statement. Define quantitatively what you mean by “resolution”. Use words, equations and sketches in your answer. What is the physical significance of the Fried parameter  $r_0$ ? Compare large and small telescopes quantitatively. Define the conditions under which the statement in quotes above is true. (It isn’t true all the time!) What are the assumptions you are making?
- (b) Assume that this amateur telescope is an 8-inch telescope in your own back yard. Assume that the native “seeing” in your back yard is 1 arc sec FWHM at a wavelength of 0.5 microns. For what wavelength range of light would your little telescope have diffraction-limited spatial resolution? For what wavelength range would your little telescope benefit from tip-tilt correction only?

**Primack: PHYS 224 – Origin and Evolution of the Universe**

Standard principles

- (a) Derive the expression for the critical density of the universe, assuming that the Hubble parameter is 70 km/s/Mpc. Express it in units of eV per cubic centimeter.
- (b) If the cosmic microwave background (CMB) temperature is 2.7 K, what fraction of critical density is the energy in the CMB?
- (c) What was the temperature in eV at redshift  $z=1000$ , when recombination occurred? Compare this to the binding energy of a hydrogen atom, and explain why they are different.

**Bolte: ASTR 257 – Modern Observational Techniques**

- (1) What is the expression for the signal-to-noise ratio for a source measured in an aperture with radius  $r$  in a CCD image?
- (2) What is the ratio of intensities for two stars with a magnitude difference of 3.5 mag?
- (3) A  $R = 20$  star observed with the Prime Focus Camera at the Lick Observatory 3-meter telescope produces 280 photo-electrons ( $e^-$ ) per second in the  $R$  band at zenith. The  $R$ -band sky brightness at Lick is 20.3 mag/arcsec<sup>2</sup>. The PFCam pixel scale is 0.30 arcsec/pixel, the readout noise is  $5 e^-$  and the inverse gain of the system is  $2.5 e^-/\text{DN}$ .
  - (a) What is the rate of detected  $e^-/\text{pixel}$  from the sky in the  $R$  band?
  - (b) How does the exposure time to a given S/N scale with the measurement aperture radius (for the sky-limited case)?
  - (c) How would the count rate from a given star scale for an imager with the same throughput on a 5-meter telescope? How would the sky counts per pixel scale for this case if the final focal-plane scale and pixel size for the 5-meter was the same as it is for ESI?

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