

YOUR NAME: _____ 2009

ASTRONOMY & ASTROPHYSICS EXAM 2009: ADVANCED PART

This is a 3 hour exam, and you only need to answer 6 of the 16 questions. Each question is identified at the top with the instructor and course number so you can go directly to those questions that are relevant to the courses you took.

To avoid confusion and in keeping with previous policy, submit answers to only 6 questions. If you attempt to answer more than 6 questions, please cross out your work on the additional questions so that it is clear which ones you wish to submit.

As an additional confirmation of the six questions that you wish to submit, please check them in the Table below.

Please use only one side of each page for your answers. If you need to extend your answer to more than one page, continue your work on one of the additional pages supplied during the exam. Be sure to put your name on every page that you turn in and, if you need to use additional pages, add both the problem number and your name at the top of each page.

You may use a hand calculator on this exam.

ANSWERS SUBMITTED

Question	Course	Six Answers Submitted ^(a)
1	Lin: ASTR 222	_____
2	Illingworth: ASTR 207	_____
3	Illingworth: ASTR 240B	_____
4	Laughlin: ASTR 212	_____
5	Krumholz: ASTR 220B	_____
6	Woosley: ASTR 220C	_____
7	Aguirre: PHYS 226	_____
8	Madau: ASTR 240C	_____
9	Rockosi: ASTR 260	_____
10	Prochaska: ASTR 230	_____
11	Primack (Dekel): ASTR 233	_____
12	Bolte: ASTR 257	_____
13	Max: ASTR 289C	_____
14	Primack: PHYS 224	_____
15	Ramirez-Ruiz: PHYS 225	_____
16	Primack: PHYS 214	_____

^(a) Please check the six questions that you are submitting.

ASTROPHYSICS EXAM INFORMATION SHEET

Physical constants:

speed of light in vacuum	c	$2.998 \times 10^8 \text{ m/s} = 2.998 \times 10^{10} \text{ cm/s}$
Gravitational constant	G	$6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2 = 6.67 \times 10^{-8} \text{ cm}^3/\text{g s}^2$
Elementary charge	e	$1.60 \times 10^{-19} \text{ C} = 4.80 \times 10^{-10} \text{ esu}$
Planck constant	h	$6.625 \times 10^{-34} \text{ Js} = 6.625 \times 10^{-27} \text{ erg s}$
Fine structure constant	$\alpha = e^2/\hbar c$	1/137
Boltzmann constant	k	$1.38 \times 10^{-23} \text{ J/K} = 1.38 \times 10^{-16} \text{ erg/K}$
Gas constant	\mathcal{R}	$= 8.32 \times 10^7 \text{ erg K}^{-1} \text{ mole}^{-1}$
Electron mass	m_e	$9.11 \times 10^{-31} \text{ kg} = 9.11 \times 10^{-28} \text{ gm}$
Proton mass	m_p	$1836m_e$
Electron classical radius	$r_e = e^2/m_e c^2$	$2.82 \times 10^{-15} \text{ m} = 2.82 \times 10^{-13} \text{ cm}$
Compton wavelength	$h/m_e c$	$2.426 \times 10^{-12} \text{ m} = 2.426 \times 10^{-10} \text{ cm}$
Bohr radius	$a_B = \hbar^2/m_e e^2$	$0.529 \times 10^{-10} \text{ m} = 0.529 \times 10^{-8} \text{ cm}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$5.79 \times 10^{-11} \text{ MeV/T}$
Rydberg energy	$m_e c^2 \alpha^2/2$	13.6 eV
Stephan Boltzmann const.	$\sigma_{SB} = 2\pi^5 k^4/15c^2 h^3$	$5.67 \times 10^{-8} \text{ J/s m}^2 \text{ K}^4 = 5.67 \times 10^{-5} \text{ erg/s cm}^2 \text{ K}^4 \text{ s}$
radiation constant	$a = 4\sigma_{SB}/c$	$7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$
Thompson scattering	$\sigma_T = (8\pi/3)r_e^2$	$6.65 \times 10^{-29} \text{ m}^2 = 6.65 \times 10^{-25} \text{ cm}^2$
Avogadro number	N_A	$6.022 \times 10^{23} \text{ mol}^{-1}$

Astrophysical Quantities:

M_\odot	$2 \times 10^{33} \text{ g}$
L_\odot	$4 \times 10^{33} \text{ erg s}^{-1}$
R_\odot	$7 \times 10^{10} \text{ cm}$

Unit conversions:

electron volt	$1.60 \times 10^{-12} \text{ erg}$
year	$3.15 \times 10^7 \text{ s}$
Joule	10^7 erg
arc second	$4.848 \times 10^{-6} \text{ radians}$
Angstrom	10^{-8} cm
1 AU	$1.50 \times 10^{13} \text{ cm}$
parsec	$3.08 \times 10^{18} \text{ cm}$

Other information of questionable usefulness:

sound speed in air at 300° K	330 m/s	$3.30 \times 10^4 \text{ c/s}$
atmospheric pressure	$1. \times 10^5 \text{ N/m}^2$	
acceleration of gravity	9.8 m/s^2	980 cm/s^2

Equations of interest:

Maxwell's equations $\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \quad \nabla \cdot \mathbf{H} = 0$

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad \nabla \cdot \mathbf{E} = 4\pi\rho$$

ideal gas $P = \rho kT / (\mu m_p) = \rho \mathcal{R}T / \mu$

blackbody $B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$

blackbody radiation density $u = (4\sigma_{SB}/c)T^4 \equiv a_B T^4$

first law $dQ = dE + PdV$

Schrodinger's equation $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2\mu} \nabla^2 \Psi + U(x, y, z) \Psi$

$$\left(\frac{\hbar^2}{2\mu}\right) \nabla^2 \Psi + [E - U(x, y, z)] \Psi = 0$$

Friedmann's Equation $H^2 = H_0^2 \left[\frac{\Omega_M}{a^3} + \frac{\Omega_K}{a^2} + \frac{\Omega_R}{a^4} + \Omega_\Lambda \right]$

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Lin: ASTR 222 – PLANETARY SCIENCE

Define the following concepts, using a quantitative example wherever possible, and explain the relevance of each item to planet formation and/or evolution.

- (a) Dynamical Friction
- (b) Roche Radius
- (c) Core Accretion
- (d) Ambipolar Diffusion
- (e) Balbus-Hawley Instability
- (f) The Planetary Disturbing Function

Illingworth: ASTR 207 – FUTURE DIRECTIONS

You are the Astronomy Division Director at the NSF. Consider the following situation:

Its late 2010. The “Astro2010” Decadal Survey has just been released.

The Survey says that the highest priority ground-based facility is a 30-m class GSMT. It says that the US community should get 50% of such a facility. It also says to go ahead with LSST as a second priority, but expresses concern about the cost.

There is competitive pressure from Europe who expect to have 100% of their own 30-40 m Extremely Large Telescope by 2019.

The two projects TMT and GMT are clamoring for a decision and a commitment by late 2011 so they can go ahead into construction in 2012. Each can only offer 20-30% to the US community, since the rest is to be privately funded. Both TMT and GMT want to finish construction and have first light in 2018.

However, even 20-30% is still a lot of money (each telescope will cost \$800-1000M). So around \$200-300M is needed for 20-30%.

Construction on the \$260M solar telescope from the last Decadal Survey has just started after 7 years of reviews at the NSF. So it has tied up the NSF construction pool for a few years.

And LSST is pressing for its \$250M of support from the NSF (and assumes that it will get the support from DOE and private money for the other \$200M that it needs). Its proposal for \$450M of construction was submitted to the NSF in late 2007.

If LSST goes ahead it will tie up the construction pool for many more years, effectively “freezing-out” GSMT until late in the decade (2017?)

What do you do? Discuss what seems to be an approach that will help the Astronomy goals. In the course of your answer, think about:

- (a) Do you go ahead with LSST knowing that it could delay GMST?
- (b) How do you deal with having two GSMT projects? Do you think you can convince people in the NSF, OMB and Congress to fund two telescopes?
- (c) Who do you consult?
- (d) What are the criteria that you might use in your decisions?
- (e) Each telescope will need about 5-7% of its total budget to operate. How do you deal with operations?

Illingworth: ASTR 240B – GALACTIC AND EXTRAGALACTIC STELLAR SYSTEMS

In the past few years, there has been much discussion of compact, dense and relatively massive “red” galaxies at $z = 2 - 3$.

- (a) What are these galaxies, and why are they a challenge to the theoretical framework?
- (b) What do they evolve into at the present time? In your consideration of this question, remind us of where the baryons are today. That is, what fraction is in stars, and what fraction is in gas/plasma, and what fraction of the stellar mass is in the spheroids/bulges, and what is in the disk components (and additionally, what is in the small late-type galaxy population)?
- (c) What does this tell us about galaxy formation in early times?
- (d) What additional information accrues from the discovery that evolved galaxies and even a rudimentary “red” sequence exist at $z \sim 2 - 2.5 - 3$? What role do ULIRGS or sub-mm galaxies play at these redshifts?
- (e) What changes do you think we will see in such populations (evolved, dusty, ULIRG/sub-mm galaxies) when we look much more extensively at $z \sim 4 - 6$ and even $z \sim 7 - 8$? (Hint - think about the time elapsed since recombination, when reionization ended and the $\Delta(\text{time})$ elapsed from $z \sim 6$ to $z \sim 3$).
- (e) What are the issues that are a major challenge for our current galaxy formation/evolution models?

Laughlin: ASTR 212 – DYNAMICAL ASTRONOMY

Consider the radially symmetric potential

$$\Phi(r) = \frac{1}{2}\Omega^2 r^2 + \text{constant} \quad (1)$$

- (a) Show that the orbital angular momentum of a particle moving in this potential is conserved.
- (b) Using cartesian coordinates, defined by $x = r \cos \phi$, $y = r \sin \phi$, write down the equations of motion of a particle moving in the x - y plane.
- (c) Write down the general analytic solution for the motion, and give a physical interpretation of the constants of integration.
- (d) Returning to polar coordinates, what are the periods of angular and radial oscillation? What is the epicyclic frequency?

Krumholz: ASTR 220B – STAR AND PLANET FORMATION

A Simple Derivation of HII Region Expansion. Consider a cloud of pure hydrogen with number density of hydrogen nuclei n_0 . At time $t = 0$ a massive star begins to shine somewhere within it with an ionizing luminosity S (in units of ionizing photons per second).

- Calculate the radius of the region that is ionized by the star once the ionization radius reaches its equilibrium value, but before the gas density begins to change. You may use the “on-the-spot” approximation.
- The ionized region is heated up so that its sound speed c_i greatly exceeds the sound speed in the surrounding neutral gas. As a result, the ionized region expands, sweeping up a shell of neutral material. Let r_{sh} and M_{sh} be the radius and mass of the shell. Write an equation describing the time evolution of the momentum of the shell. You may neglect the pressure outside the shell, the force of gravity, and the mass (but not the density) in the shell interior, and you may assume that the expansion velocity v_{sh} is much smaller than c_i .
- Your equation from part (b) should admit a solution of the form $r_{\text{sh}} = at^\eta$. Plug this in and solve for a and η to obtain r_{sh} as a function of time.

Woosley: ASTR 220C – ADVANCED STAGES OF STELLAR EVOLUTION

- (a) Describe the principal nuclear reactions involved in neon burning. Why is it that neon burns before oxygen, even though oxygen has a smaller nuclear charge?
- (b) What is a detonation front? A deflagration front? As matter crosses each kind of front what happens (increase, decrease, or constant) in each case to the: i) pressure, ii) temperature, and iii) density?
- (c) Describe the three stages of the light curve of a Type IIp supernova. Draw a sketch giving approximate time scales for each (and luminosities if you remember them). What is the source of energy in each stage?

Aguirre: PHYS 226 – GENERAL RELATIVITY

It's nice when familiar things fall out of the big machinery of GR. Consider plain old Minkowski space with metric

$$ds^2 = -c^2 dT^2 + dR^2 + R^2 d\phi^2 + dZ^2$$

in cylindrical coordinates. Now let's look at a rotating frame given by

$$t = T, \quad r = R, \quad \theta = \phi - \omega T, \quad z = Z.$$

This gives metric

$$ds^2 = - \left(1 - \frac{r^2 \omega^2}{c^2} \right) c^2 dt^2 + dr^2 + 2r^2 \omega dt d\theta + r^2 d\theta^2 + dz^2.$$

1. Using the formula for the Christoffel symbols,

$$\Gamma_{\alpha\beta}^{\mu} = \frac{1}{2} g^{\mu\nu} (g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu}),$$

compute the Christoffel symbols Γ_{tt}^r for this metric (note that you don't have to do the matrix inversion to do this, you just have to know that as it happens, $g^{rr} = g_{rr}^{-1}$.)

2. Use this, the other nonzero Christoffel symbols, $\Gamma_{\theta\theta}^r = -r$, $\Gamma_{t\theta}^r = -\omega r$, and $\Gamma_{\theta r}^{\theta} = \frac{1}{r}$, $\Gamma_{tr}^{\theta} = \omega/r$, and the geodesic equation, to find an equation for $d^2 r / d\tau^2$ (where τ is the proper time) for a particle following a geodesic
3. Interpret your equation in the non-relativistic limit.

Madau: ASTR 240C – GALACTIC & EXTRAGALACTIC STELLAR SYSTEMS

Given a population of objects with proper number density $n(z)$ and cross section $\sigma(z)$, the differential probability dP that a line of sight will intersect one of the objects in the redshift interval $(z, z + dz)$ is

$$dP = n(z)\sigma(z)adr = n_s(z)\sigma(z)|cdt/dz|dz = n(z)\sigma(z)\frac{c}{H(z)(1+z)}dz,$$

where $H(z)$ is the Hubble parameter. For ordinary galaxies, $n_0 \sim 0.02h^3 \text{ Mpc}^{-3}$ and the cross-section for the bright part is $\sigma \sim \pi(10h^{-1} \text{ kpc})^2$. Assuming galaxies do not evolve with redshift, compute the total optical depth to redshift z in an Einstein-de Sitter Universe. What is the fraction of the sky covered by galaxies at redshift 2?

Rockosi: ASTR 260 – INSTRUMENTATION FOR ASTRONOMY

You build a wide-field, fast, re-imaging camera for the Lick 3m telescope at the f/17 Cassegrain focus. That is, you design and build an optical system that makes an image of the field of view of the 3-m telescope such that the final scale of the image, in arcsec/mm, is the same as if the focal ratio of the 3m were f/5.

The CCD camera that you use as the detector for your new camera has 15 micron pixels and a readnoise of 5 electrons. Before it went into your camera, the same CCD was used at the f/17 focal plane of the 3m. For sky brightness in the u filter of 22.1 magnitudes per square arcsecond, you got 0.32 counts per second per pixel when using the camera at the f/17 focus.

- (a) In a 60-second exposure at the f/17 focus, was the total noise in each pixel dominated by the sky or the detector?
- (b) How many counts per second per pixel do you expect with your new camera (you can ignore any additional losses from the optics of your camera).
- (c) Your goal in designing the camera was to be sky-noise dominated in the u band for a 60-second exposure. Was f/5 a good choice?
- (d) Sketch the optical layout of your camera.

Prochaska: ASTR 230 – Low Density Astrophysics

The collisional de-excitation rate may be expressed as:

$$q_{ji} = \frac{C\Omega_{ij}}{g_j T^{\frac{1}{2}}} e^{-(E_j - E_i)/kT}$$

where $C = 8.63 \times 10^{-6} \text{cm}^3 \text{K}^{\frac{1}{2}} \text{s}^{-1}$ and Ω_{ij} is the collisional strength. Also recall the excitation rate $q_{ij} = (g_j/g_i)q_{ji} \exp(-\Delta E/kT)$.

The ratio of the emission line strengths from the two levels just above the true ground state provides a useful density diagnostic of HII regions.

- (a) Estimate the value of this ratio in the low and high density limits.
- (b) Explain why this ratio is a useful density diagnostic and not a useful temperature diagnostic.

Helpful information: Recall the emission line strength j is the energy/s/volume. Assume the spontaneous coefficients for the two upper levels (2,3) to the ground (1) are $A_{21} = 3.6 \times 10^{-5} \text{s}^{-1}$ and $A_{31} = 1.6 \times 10^{-4} \text{s}^{-1}$ where $\lambda_{21} > \lambda_{31}$. You should also find the following equation to be useful:

$$\Omega(S'L'J', SLJ) = \frac{(2J+1)}{(2S+1)(2L+1)} \Omega(S'L', SL)$$

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Primack: ASTR 233 – Physical Cosmology

Suppose astronomers measure the age of a galaxy at redshift $z = 3$. How old would this galaxy have to be (at the time its light was emitted) in order to rule out the hypothesis that $\Omega_m = 1$ with negligible vacuum and radiation energy density? Use $H_0 = 70$ km/sec/Mpc.

Bolte: ASTR 257 – MODERN OBSERVATIONAL TECHNIQUES

- (a) What is the ratio of brightness (measured flux) in the V band for two stars if one is $V = 21$ and the second $V = 18.2$?
- (b) What is the general expression for the signal-to-noise ratio for a source measured in an aperture with radius r in a CCD image? (be sure to define each term, including units).

An $R = 23$ star observed with the Prime Focus Camera at the Lick 3m produces 80 photoelectrons per second in the R band at zenith. The R -band sky brightness at Lick is $20.3 \text{ mag/arcsec}^2$. The PFCam pixel scale is $0.30 \text{ arcseconds/pixel}$, the readout noise is $5e^-$ and the inverse gain of the system is $2.5 e^-/\text{DN}$.

- (i) What is the noise due to the sky in a 10-pixel diameter circular aperture?
- (ii) What is the S/N measured in a 10-pixel diameter circular aperture in 600 seconds?

Max: ASTR289C – Adaptive Optics

This problem explores the point spread function (PSF).

- (a) Define the point spread function of an optical system in at least two ways.
- (b) Define the Modulation Transfer function (MTF). Explain how it is related to the point-spread function.
- (c) Draw a rough sketch of the PSF produced by an astronomical AO system with Strehl of 70% in a long-exposure image. Label all important features. Indicate on your sketch expressions for the approximate dimensions of the main features in the PSF. Draw the corresponding MTF and indicate the important features and scales on your sketch.
- (d) Do the same for Strehl ratio of 5%.
- (e) Draw a sketch of the effects of image motion on the PSF in part (c) above.

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PHYS 224 – PHYSICAL COSMOLOGY – Joel Primack

Calculate the redshift of matter-radiation equality, z_{eq} , in terms of Ω_m and h . You may assume that the photon blackbody temperature today is $T_\gamma = 2.73 K$, and that the total energy density in radiation, i.e. relativistic particles, is (for photons plus three massless neutrino species) $\rho_r = 1.68\rho_\gamma$, where ρ_γ is the photon energy density.

PHYS 225 – COMPACT OBJECTS – Enrico Ramirez-Ruiz

General relativity predicts that accelerated masses radiative gravitational waves, thereby losing energy, in analogy to the emission of electromagnetic radiation by accelerated charges.

[a] If gravitational radiation were responsible for the loss of rotational energy E_{rot} of isolated pulsars (e.g. the Crab), then a dependence $dE_{\text{rot}}/dt \propto \omega^6$ would be expected, where ω is the angular velocity. Under the above assumption derive an upper limit for the age of the Crab pulsar. Given that the supernova that marked the Crab's formation occurred in the year 1054, is gravitational radiation a viable braking mechanism?

[b] A type Ia supernova is thought to be the thermonuclear explosion of an accreting white dwarf that goes over the Chandrasekhar limit. An alternative scenario, however, is that supernova Ia progenitors are white dwarfs binaries, that lose orbital energy to gravitational waves until they merge, and thus exceed the Chandrasekhar mass and explode. What is the maximum initial separation that a white-dwarf binary can have, if the components are to merge within 10 Gyr? Assume the white dwarfs have $1M_{\odot}$ each, and that the merger occurs at $a = 0$.

Hint: the power lost to gravitational radiation by such a system is

$$\dot{E}_{\text{gw}} = \frac{-2c^5}{5G} \left(\frac{2GM}{c^2 a} \right)^5.$$

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PHYS 214 – STRUCTURE FORMATION IN THE UNIVERSE – Joel Primack

Guaranteeing a Big Bang

Show that measurement of $\Omega_m > 0.01$ and the observation of objects at $z > 5$ guarantee there was a Big Bang, regardless of any cosmological constant.

Hint: show that in the past $a \rightarrow 0$ by considering \dot{a} and \ddot{a} in models with matter and an arbitrary amount of the most “anti-Big Bang” component, Λ .

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