YOUR NAME: ______________________________ 2006

ASTRONOMY & ASTROPHYSICS EXAM 2006: BASIC PART

This is a **2 hour** exam, with 9 questions. It is required that *all* questions be answered. Each question is identified at the top with the course number. Please use only one side of each page for your answers. If you need to extend your answer to more than one page, continue your work on one of the additional pages supplied during the exam. Be sure to put your name on every page that you turn in and, if you need to use additional pages, add both the problem number and your name at the top of each page.

You may use a hand calculator on this exam.

ASTROPHYSICS EXAM INFORMATION SHEET

Physical constants:

speed of light in vacuum	c	2.998×10^8 m/s $= 2.998 \times 10^{10}$ cm/s
Gravitational constant	G	6.67×10^{-11} m^3/kg s^2 $= 6.67 \times 10^{-8}$ cm^3/g s^2
Elementary charge	e	1.60×10^{-19} C $= 4.80 \times 10^{-10}$ esu
Planck constant	h	6.625×10^{-34} Js $= 6.625 \times 10^{-27}$ erg s
Fine structure constant	$\alpha = e^2/\hbar c$	1/137
Boltzmann constant	k	1.38×10^{-23} J/K $= 1.38 \times 10^{-16}$ erg/K
Gas constant	$\mathcal{R}$	$= 8.32 \times 10^7$ erg K^{-1} mole^{-1}
Electron mass	m_e	9.11×10^{-31} kg $= 9.11 \times 10^{-28}$ gm
Proton mass	m_p	$1836 m_e$
Electron classical radius	$r_e = e^2/m_e c^2$	2.82×10^{-15} m $= 2.82 \times 10^{-13}$ cm
Compton wavelength	$h/m_e c$	2.426×10^{-12}m $= 2.426 \times 10^{-10}$cm
Bohr radius	$a_B = \hbar^2/m_e e^2$	0.529×10^{-10} m $= 0.529 \times 10^{-8}$ cm
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.79×10^{-11} MeV/T
Rydberg energy	$m_e c^2 \alpha^2/2$	13.6 eV
Stephan Boltzmann const.	$\sigma_{SB} = 2\pi^5 k^4/15c^2h^3$	5.67×10^{-8} J/s m^2 K^4 $= 5.67 \times 10^{-5}$ erg/s cm^2 K^4 s
radiation constant	$a = 4\sigma_{SB}/c$	7.56×10^{-15} erg cm^{-3} K^{-4}
Thompson scattering	$\sigma_T = (8\pi/3) r_e^2$	6.65×10^{-29} m^2 $= 6.65 \times 10^{-25}$ cm^2
Avogadro number	N_A	6.022×10^{23} mol^{-1}

Astrophysical Quantities:

$M_\odot$	2×10^{33} g
$L_\odot$	4×10^{33} erg s^{-1}
$R_\odot$	7×10^{10} cm

Unit conversions:

electron volt	1.60×10^{-12} erg
year	3.15×10^7 s
Joule	10^7 erg
arc second	4.848×10^{-6} radians
Angstrom	10^{-8} cm
1 AU	1.50×10^{13} cm
parsec	3.08×10^{18} cm

Other useful information:

sound speed in air at 300 K	330 m/s	3.30×10^4 c/s
atmospheric pressure	$1. \times 10^5$ N/m^2	
acceleration of gravity	9.8 m/s^2	980 cm/s^2

Equations of interest:

Maxwell's equations	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \cdot \mathbf{B} = 0$
	$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} \qquad \nabla \cdot \mathbf{D} = 4\pi\rho$
ideal gas	$P = \rho kT/(\mu m_p) = \rho \mathcal{R} T/\mu$
blackbody	$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}$
blackbody radiation density	$u = (4\sigma_{SB}/c)T^4 \equiv a_B T^4$
first law	$dQ = dE + PdV$
Schrodinger's equation	$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2\mu}\nabla\Psi + U(x,y,z)\Psi$
	$(\frac{\hbar^2}{2\mu})\nabla\Psi + [E - U(x,y,z)]\Psi = 0$
Friedmann's Equation	$H^2 = H_0^2\left[\frac{\Omega_M}{a^3} + \frac{\Omega_K}{a^2} + \frac{\Omega_R}{a^4} + \Omega_\Lambda\right]$
useful Lorentz invariants	P, dn/ν, $u^\mu u_\mu$, $\partial_\mu A^\mu = \frac{1}{c}\frac{\partial\phi}{\partial t} + \nabla\cdot\mathbf{A}$
	$F^{\mu\nu}F_{\mu\nu} = 2(B^2 - E^2)$

1. **A220A: STELLAR STRUCTURE AND EVOLUTION**

   Energy transport in stars can occur by radiation, conduction, and convection . For the various phases of the evolution of a 1 solar mass star, (pre-main-sequence, main sequence, post main sequence, final state) identify which of these mechanisms is significant for energy transport and roughly in which region of the star it occurs. Compare with the evolution of a star of 5 solar masses.

2. **A220A: STELLAR STRUCTURE AND EVOLUTION**

Compare the astrophysical concept of *local thermodynamic equilibrium* (LTE) with that of *thermodynamic equilibrium*. What physical relations hold in both? What physical relations are different? What criterion needs to be satisfied to ensure that LTE holds? Give an example of an astrophysical environment where LTE is valid and one where is isn't.

3. **A220A: STELLAR STRUCTURE AND EVOLUTION**

State the Virial Theorem in a form appropriate for a main-sequence star. Given that the mean density of a star is $\rho = 3M/(4\pi R^3)$, and that the equation of state is that of an ideal gas, obtain the mean pressure as a function of mass M and radius R. Show that this result is dimensionally correct. Now do the same for the case where the equation of state is radiation-pressure dominated. Can you explain the difference?

4. **240A Galactic and Extragalactic Stellar Systems**

Assume that the Universe is exactly closed and that it is normal matter-dominated, i.e., that it has critical density (ignore dark matter and radiation). Assume that the density of matter is everywhere uniform at all epochs.

(a) Show that the scale factor of the Universe, a, varies as $t^{2/3}$.

(b) Recall that $a \propto 1/(1+z)$. If the age of the Universe now is 13.5 Gyr, how old was it at $z = 1$, assuming that it expanded following the rule in part (a)?

(c) If the matter density now is 10^{-29} gm/cc, what was the density at $z = 1$?

(d) The real Universe has less than critical density. Does the answer in part (c) need revision on that account?

5. **240A Galactic and Extragalactic Stellar Systems**

   This is a question about element abundance ratios in old stars.

   (a) Describe the two types of supernovae that produce heavy elements via nucleosynthesis. What are the main sequence masses of the progenitor stars? What triggers the explosions in the two cases? What elements are produced by the two types?

   (b) It is observed that old stars in our Galaxy have higher ratios of O to Fe than younger stars like the Sun. What is the leading theory to explain this?

   (c) What is known about element abundance ratios in elliptical galaxies, and what might they say about the star-formation histories of these galaxies?

6. **240A Galactic and Extragalactic Stellar Systems**

This problem tests your understanding of errors and how to combine them properly to get a final error. All errors below are Gaussian rms errors.

(a) An observer has measured the radius (r) and rotation speed (v) of a galaxy and wishes to calculate the mass M using the formula $M = v^2 r/G$. The data are:

$$v = 200 \pm 10 \text{ km/s} \qquad r = 10 \pm 0.5 \text{ kpc}.$$

The errors in v and r are uncorrelated. What is the percentage error in the mass?

(b) The same observer now wants to calculate the luminosity of the same galaxy. She knows the distance d and the apparent magnitude. The distance is:

$$d = 100 \pm 5 \text{ Mpc},$$

and the apparent magnitude m_{app} is uncertain by 0.1 mag. What is the error in the absolute mag, M_{abs}? (Hint: in this part the aim is to get an error estimate quickly using the rough rule of thumb between delta (mag) and error.)

7. **297/202/PHYS213 Electromagnetism and Plasma Physics**

A new elementary particle, the spiton, is found to be unstable when it experiences a nonzero electrostatic potential. In particular, in its own rest frame (denoted by a prime), the spiton decays at a rate given by

$$\frac{dN'}{dt'} = k\phi'$$

where k is a constant and ϕ' is the electric potential in the particle's rest frame. The decay rate is independent of the electric field, the magnetic field, or the vector potential. Find a Lorentz covariant expression for the decay rate of spitons, which can be written as a scalar. [Some possibly useful four-vectors and tensors are shown in the equations page of this exam.]

8. **297/202/PHYS213 Electromagnetism and Plasma Physics**

Inside of a supernova remnant, the ambient radiation is isotropic and has the energy spectrum

$$du = k\nu^{-1} d\Omega d\nu \quad \text{for} \quad \nu_a < \nu < \nu_b$$

where k, ν_a, and ν_b are constants and du is the energy density within frequency interval $d\nu$ and solid angle $d\Omega$. Find the spectrum and angular distribution of the radiation seen by a relativistic particle traversing the remnant at speed $c\beta$.

Note the following useful relations:

$$\nu = \gamma\nu'(1 + \beta\cos\theta') \qquad \text{Doppler}$$

$$\cos\theta = \frac{\cos\theta' + \beta}{1 + \beta\cos\theta'} = \int d\cos\theta' \frac{1}{\gamma^2(1 + \beta\cos\theta')^2} \qquad \text{aberration}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \qquad \text{Lorentz factor}$$

P and dn/ν are Lorentz invariants

9. **297/202/PHYS213 Electromagnetism and Plasma Physics**

For this problem, it is helpful to know that $v_a = \sqrt{B^2/4\pi\rho}$.

(a) It is sometimes said that $p_\perp/B^2$ is an adiabatic invariant. Very briefly explain what that means.

(b) Very briefly explain what is an Alfven wave.

(c) It is sometimes said that Alfven waves carry energy from the interior of the sun into solar active regions. If an active region has a magnetic field of $B \sim 100$ gauss and an ionized gas density of $\rho \sim 10^{-13}$ grams/cm^3, **estimate** the maximum energy flux (energy per unit time per unit area) that Alfven waves can carry. Assume that the field associated with the wave cannot exceed the ambient field.

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