

YOUR NAME: \_\_\_\_\_ 2005

ASTRONOMY & ASTROPHYSICS EXAM 2005: BASIC PART

This is a 1.5 hour exam, with 9 questions. It is required that \*all\* questions be answered. Each question is identified at the top with the course number. Please use only one side of each page for your answers. If you need to extend your answer to more than one page, continue your work on one of the additional pages supplied during the exam. Be sure to put your name on every page that you turn in and, if you need to use additional pages, add both the problem number and your name at the top of each page.

You may use a hand calculator on this exam.

## ASTROPHYSICS EXAM INFORMATION SHEET

### Physical constants:

speed of light in vacuum	$c$	$2.998 \times 10^8 \text{ m/s} = 2.998 \times 10^{10} \text{ cm/s}$
Gravitational constant	$G$	$6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2 = 6.67 \times 10^{-8} \text{ cm}^3/\text{g s}^2$
Elementary charge	$e$	$1.60 \times 10^{-19} \text{ C} = 4.80 \times 10^{-10} \text{ esu}$
Planck constant	$h$	$6.625 \times 10^{-34} \text{ Js} = 6.625 \times 10^{-27} \text{ erg s}$
Fine structure constant	$\alpha = e^2/\hbar c$	1/137
Boltzmann constant	$k$	$1.38 \times 10^{-23} \text{ J/K} = 1.38 \times 10^{-16} \text{ erg/K}$
Gas constant	$\mathcal{R}$	$= 8.32 \times 10^7 \text{ erg K}^{-1} \text{ mole}^{-1}$
Electron mass	$m_e$	$9.11 \times 10^{-31} \text{ kg} = 9.11 \times 10^{-28} \text{ gm}$
Proton mass	$m_p$	$1836m_e$
Electron classical radius	$r_e = e^2/m_e c^2$	$2.82 \times 10^{-15} \text{ m} = 2.82 \times 10^{-13} \text{ cm}$
Compton wavelength	$h/m_e c$	$2.426 \times 10^{-12} \text{ m} = 2.426 \times 10^{-10} \text{ cm}$
Bohr radius	$a_B = \hbar^2/m_e e^2$	$0.529 \times 10^{-10} \text{ m} = 0.529 \times 10^{-8} \text{ cm}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$5.79 \times 10^{-11} \text{ MeV/T}$
Rydberg energy	$m_e c^2 \alpha^2/2$	13.6 eV
Stephan Boltzmann const.	$\sigma_{SB} = 2\pi^5 k^4/15c^2 h^3$	$5.67 \times 10^{-8} \text{ J/s m}^2 \text{ K}^4 = 5.67 \times 10^{-5} \text{ erg/s cm}^2 \text{ K}^4 \text{ s}$
radiation constant	$a = 4\sigma_{SB}/c$	$7.56 \times 10^{-15} \text{ erg cm}^{-3} \text{ K}^{-4}$
Thompson scattering	$\sigma_T = (8\pi/3)r_e^2$	$6.65 \times 10^{-29} \text{ m}^2 = 6.65 \times 10^{-25} \text{ cm}^2$
Avogadro number	$N_A$	$6.022 \times 10^{23} \text{ mol}^{-1}$

### Astrophysical Quantities:

$M_\odot$	$2 \times 10^{33} \text{ g}$
$L_\odot$	$4 \times 10^{33} \text{ erg s}^{-1}$
$R_\odot$	$7 \times 10^{10} \text{ cm}$

### Unit conversions:

electron volt	$1.60 \times 10^{-12} \text{ erg}$
year	$3.15 \times 10^7 \text{ s}$
Joule	$10^7 \text{ erg}$
arc second	$4.848 \times 10^{-6} \text{ radians}$
Angstrom	$10^{-8} \text{ cm}$
1 AU	$1.50 \times 10^{13} \text{ cm}$
parsec	$3.08 \times 10^{18} \text{ cm}$

### Other useful information:

sound speed in air at 300° K	330 m/s	$3.30 \times 10^4 \text{ c/s}$
atmospheric pressure	$1. \times 10^5 \text{ N/m}^2$	
acceleration of gravity	$9.8 \text{ m/s}^2$	$980 \text{ cm/s}^2$

## Equations of interest:

Maxwell's equations	$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \quad \nabla \cdot \mathbf{H} = 0$ $\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad \nabla \cdot \mathbf{E} = 4\pi\rho$
ideal gas	$P = \rho kT / (\mu m_p) = \rho \mathcal{R}T / \mu$
blackbody	$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$
blackbody radiation density	$u = (4\sigma_{SB}/c)T^4 \equiv a_B T^4$
first law	$dQ = dE + PdV$
Schrodinger's equation	$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2\mu} \nabla^2 \Psi + U(x, y, z) \Psi$ $\left(\frac{\hbar^2}{2\mu}\right) \nabla^2 \Psi + [E - U(x, y, z)] \Psi = 0$
Friedmann's Equation	$H^2 = H_0^2 \left[ \frac{\Omega_M}{a^3} + \frac{\Omega_K}{a^2} + \frac{\Omega_R}{a^4} + \Omega_\Lambda \right]$

**1. A220A: STELLAR STRUCTURE AND EVOLUTION**

A main-sequence star of 2 solar masses has an effective temperature of  $9 \times 10^3$  K and a luminosity of  $16 L_{\odot}$ .

- a) What is the approximate time for the star to reach the main sequence?
- b) What is the energy source? Assume that this source produces 25 MeV per helium atom produced. What is the approximate lifetime of the star on the main sequence?
- c) What principal assumptions did you use to get the times in parts a) and b)?

**2. A220A: STELLAR STRUCTURE AND EVOLUTION**

Two of the equations of stellar structure are

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

and

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\bar{\kappa}\rho}{T^3} \frac{L(r)}{4\pi r^2}.$$

Explain briefly what these equations mean. Combine these two equations to get an equation for  $dT/dP$ . Assume that the mass absorption coefficient  $\bar{\kappa} = 0.2(1 + X)$ , i.e. pure electron scattering. Now evaluate the expression in the surface layers of a star where  $L(r) = L$ , the total luminosity, and  $M(r) = M$ , the total mass, both known parameters. You should now have a clean equation relating temperature with pressure, all other quantities being constants. Integrate this equation inwards from the surface, starting at  $P = T = 0$ , to get  $P(T)$ . Assume the equation of state is ideal gas. Test the  $P(T)$  relation to see whether it is stable or unstable to convection.

**3. A220A: STELLAR STRUCTURE AND EVOLUTION**

Brown dwarfs

- a) What physically determines the boundary line between a star and a brown dwarf (defined as an object which can never supply 100% of its luminosity through nuclear burning)? That is, explain what happens to objects on both sides of this limit as they contract and approach the main sequence.
- b) Suppose the opacity in the atmosphere of objects near the borderline were reduced by a factor 100. Would the mass at the boundary (usually quoted to be  $0.075 M_{\odot}$ ), increase or decrease? Why?

**4. 240A Galactic and Extragalactic Stellar Systems**

An E0 elliptical galaxy is seen to have a half-light radius of 5 arc minutes, a surface brightness of  $\mu = 19.6$  visual magnitudes per square arc second, a redshift of 1400 km/s, and a velocity dispersion  $\sigma = 300$  km/s.

- (a) Estimate the distance and radius of the galaxy in parsecs.
- (b) Calculate  $L$  for the galaxy (in the  $V$ -band) knowing that  $M_{V_{\odot}} = 4.74$ .
- (c) Estimate  $M/L$  for the galaxy in the  $V$ -band.

**5. 240A Galactic and Extragalactic Stellar Systems**

Briefly describe

- (a) Faber-Jackson relation
- (b) Schmidt Law
- (c) Fundamental Plane
- (d) The Toomre Q
- (e) Malmquist Bias

**6. 240A Galactic and Extragalactic Stellar Systems**

Consider an open cluster (radius 2pc) where the most massive star is the only star in the 4-5 solar mass range. Furthermore, it is just leaving the main sequence and there never was a more massive star in the cluster.

- (a) Assuming a Salpeter law with no brown dwarfs, estimate the number of stars in the cluster, along with the total cluster mass.
- (b) Estimate the crossing time for a star in the cluster.
- (c) Is the cluster relaxed?

7. **297/202/PHYS213 Electromagnetism and Plasma Physics**

The velocity of a particle of charge  $e$  decreases from  $v_0$  to zero in a time interval  $T$ .

- (a) Find the angular distribution of the radiated *energy*  $E$  emitted during this time interval, assuming that the deceleration is constant. (You may assume that  $v_0 \ll c$ .)
- (b) Integrate over angles to determine the total radiated energy emitted during the deceleration.

## 8. 297/202/PHYS213 Electromagnetism and Plasma Physics

Faraday Rotation: The dispersion relations for right-handed and left-handed circularly polarized electromagnetic waves propagating parallel to a magnetic field  $B$  are

$$\text{R wave: } \frac{c^2}{v_{phase}^2} = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 - (\Omega_c/\omega)}$$

$$\text{L wave: } \frac{c^2}{v_{phase}^2} = \frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2/\omega^2}{1 + (\Omega_c/\omega)}$$

This shows that waves of the two polarizations travel at different phase velocities along the magnetic field  $B$ . Therefore a plane-polarized wave, which is a linear superposition of an R and an L circularly polarized wave, will have a plane of polarization which rotates as the wave propagates.

a) Assuming that  $\omega \gg \omega_p$  and  $\omega \gg \Omega_c$ , derive an expression for the accumulated phase-angle difference between the R and L waves  $\Delta\theta_{R,L} = \int_0^d (k_R - k_L) ds$  in terms of  $\Omega_c$  and  $\omega_p$ .

b) In practice a plane-polarized wave is rotated through an angle  $\Delta\theta_{plane} \approx \Delta\theta_{R,L}/2$ . Derive an expression for  $\Delta\theta_{plane}$  in terms of basic quantities such as  $B$ ,  $e$ ,  $n_e$ , and  $\omega$ .

9. **297/202/PHYS213 Electromagnetism and Plasma Physics**

What is a plasma?

- a) Give one or more definitions of a plasma.
- b) Describe the conditions under which a hot gas acts as a plasma.

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