ASTRONOMY & ASTROPHYSICS EXAM 2007: BASIC PART

This is a 1.5 hour exam, with 9 questions. It is required that *all* questions be answered. Each question is identified at the top with the course number. Please use only one side of each page for your answers. If you need to extend your answer to more than one page, continue your work on one of the additional pages supplied during the exam. Be sure to put your name on every page that you turn in and, if you need to use additional pages, add both the problem number and your name at the top of each page.

You may use a hand calculator on this exam.

Physical constants:

speed of light in vacuum	c	2.998×10^8 m/s $= 2.998 \times 10^{10}$ cm/s
Gravitational constant	G	6.67×10^{-11} m^3/kg s^2 $= 6.67 \times 10^{-8}$ cm^3/g s^2
Elementary charge	e	1.60×10^{-19} C $= 4.80 \times 10^{-10}$ esu
Planck constant	h	6.625×10^{-34} Js $= 6.625 \times 10^{-27}$ erg s
Fine structure constant	$\alpha = e^2/\hbar c$	1/137
Boltzmann constant	k	1.38×10^{-23} J/K $= 1.38 \times 10^{-16}$ erg/K
Gas constant	$\mathcal{R}$	$= 8.32 \times 10^7$ erg K^{-1} mole^{-1}
Electron mass	m_e	9.11×10^{-31} kg $= 9.11 \times 10^{-28}$ gm
Proton mass	m_p	$1836 m_e$
Electron classical radius	$r_e = e^2/m_e c^2$	2.82×10^{-15} m $= 2.82 \times 10^{-13}$ cm
Compton wavelength	$h/m_e c$	2.426×10^{-12}m $= 2.426 \times 10^{-10}$cm
Bohr radius	$a_B = \hbar^2/m_e e^2$	0.529×10^{-10} m $= 0.529 \times 10^{-8}$ cm
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.79×10^{-11} MeV/T
Rydberg energy	$m_e c^2 \alpha^2/2$	13.6 eV
Stephan Boltzmann const.	$\sigma_{SB} = 2\pi^5 k^4/15 c^2 h^3$	5.67×10^{-8} J/s m^2 K^4 $= 5.67 \times 10^{-5}$ erg/s cm^2 K^4 s
radiation constant	$a = 4\sigma_{SB}/c$	7.56×10^{-15} erg cm^{-3} K^{-4}
Thompson scattering	$\sigma_T = (8\pi/3) r_e^2$	6.65×10^{-29} m^2 $= 6.65 \times 10^{-25}$ cm^2
Avogadro number	N_A	6.022×10^{23} mol^{-1}

Astrophysical Quantities:

$M_\odot$	2×10^{33} g
$L_\odot$	4×10^{33} erg s^{-1}
$R_\odot$	7×10^{10} cm

Unit conversions:

electron volt	1.60×10^{-12} erg
year	3.15×10^7 s
Joule	10^7 erg
arc second	4.848×10^{-6} radians
Angstrom	10^{-8} cm
1 AU	1.50×10^{13} cm
parsec	3.08×10^{18} cm

Other useful information:

sound speed in air at 300^o K	330 m/s	3.30×10^4 c/s
atmospheric pressure	$1. \times 10^5$ N/m^2	
acceleration of gravity	9.8 m/s^2	980 cm/s^2

Equations of interest:

Maxwell's equations	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \qquad \nabla \cdot \mathbf{H} = 0$
	$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \cdot \mathbf{E} = 4\pi\rho$
ideal gas	$P = \rho kT/(\mu m_p) = \rho \mathcal{R} T/\mu$
blackbody	$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}$
blackbody radiation density	$u = (4\sigma_{SB}/c)T^4 \equiv a_B T^4$
first law	$dQ = dE + PdV$
Schrodinger's equation	$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2\mu}\nabla\Psi + U(x,y,z)\Psi$
	$(\frac{\hbar^2}{2\mu})\nabla\Psi + [E - U(x,y,z)]\Psi = 0$
Friedmann's Equation	$H^2 = H_0^2\left[\frac{\Omega_M}{a^3} + \frac{\Omega_K}{a^2} + \frac{\Omega_R}{a^4} + \Omega_\Lambda\right]$

1. A220A: STELLAR STRUCTURE AND EVOLUTION

1. A star of 1.6 solar masses has a radius of 1.55 $R_\odot$ and $T_{\rm eff} = 7000$ K at the zero-age main sequence.

a) How long, approximately, did it take for the star to reach the main sequence? This time is primarily determined by the radiative contraction phase.

b) An approximate mass-luminosity relation can be derived for equilibrium stars:

$$L \propto \frac{\mu^{7.5} M^{5.5}}{\kappa_0 R^{0.5}}$$

What assumptions go into the derivation of this formula? Suppose the opacity constant κ_0 in the star were reduced by a factor 2. What would happen to the result in part a)?

c) Would the radius at the zero-age main sequence be larger or smaller in case b) as compared with case a)? Why?

2. A220A: STELLAR STRUCTURE AND EVOLUTION

2. A problem with two parts:

a) What is the physical meaning of the following equation? What is the main assumption used in its derivation?

$$F_{\rm rad} = -\frac{4ac}{3\kappa_R \rho} T^3 \frac{dT}{dr}$$

b) How is the Eddington limit defined? Use the equation for $F_{\rm rad}$ to derive the Eddington limit luminosity $L_{\rm ed}$ in terms of the mass of the star.

3. A220A: STELLAR STRUCTURE AND EVOLUTION

3. The rare light element lithium is destroyed in stars by the reaction $^1\mathrm{H} +^7 \mathrm{Li} \rightarrow^4 \mathrm{He} +^4 \mathrm{He}$ at temperatures above about 3×10^6 K.

a) How is ^{7}Li made?

b) Assuming that Li is burned only in the interior of a star, how do observers know that it has been destroyed?

c) The abundance of Li in meteorites is 100 times larger than that in the photosphere of the Sun. How is this fact explained?

d) How is ^{7}Li used as a test to prove that an object is a brown dwarf?

4. **240A Galactic and Extragalactic Stellar Systems**

1) A galaxy has a molecular absorption feature that must be measured to high accuracy. The feature is 10% deep and 10 Å wide, and you can assume for simplicity that it is flat across the bottom.

(a) What is the equivalent width (EW) of the feature?

(b) A spectrum of the object is taken at a dispersion of 1 Å per pixel. How many counts per pixel are required in order to measure the EW of the feature with a 1-σ accuracy of 0.1 Å? Assume that the only source of noise is photon statistics in the galaxy spectrum and that the continuum can be estimated over a long wavelength interval so that the error in its estimate is negligible.

(c) Now assume that in addition to the galaxy, light from the night sky is also detected. The spectrum of the night sky is a smooth continuum near the feature, and this continuum is nine times brighter than that of the galaxy. For the same integration time as you derived in part (b), what is the 1-σ error in the EW? Assume that the error in the continuum levels of both sky and galaxy are again negligible.

5. **240A Galactic and Extragalactic Stellar Systems**

The radial Jeans equation expressing gravitational equilibrium in a spherically symmetric self-gravitating galaxy is:

$$GM(r) = [\frac{d\ln\nu}{d\ln r} - \frac{d\ln\sigma_r}{d\ln r} - 2(1 - \frac{\sigma_T^2}{\sigma_r^2})]$$

Spherically symmetric means that the mass distribution and the tracer distribution are functions of r only and that the velocity distribution function, $f(\mathbf{v})$, is a function of r, the radial component, v_r, and the transverse component v_T only.

(a) Describe in words the physical meaning of the following quantities: (1) ν, (2) $M(r)$, (3) σ_r.

(b) What is the physical meaning of each term in the above equation?

(c) Show that the above equation applied to the singular isothermal sphere simplifies to:

$$GM(r) = \gamma\sigma_r^2 r$$

where γ is a constant. Recall that the velocity ellipsoid in the isothermal sphere is spherically symmetric and assume that the tracer population and the mass-generating population are now the same. Use the fact that the radial mass density in the isothermal sphere is a power law in r.

(d) The above formula shows that $M(r)$ grows as r for the isothermal sphere. From this, deduce the value of γ.

6. **240A Galactic and Extragalactic Stellar Systems**

3. Galactic reddening: note, part (b) needs a calculator.

(a) Name three ways of estimating internal dust reddening and/or absorption within an external galaxy. Assume that the foreground Milky Way reddening and absorption are known, and assume anything you like about the stellar population, morphology and distance of the external galaxy. Thus, one method you mention might apply to nearby spiral galaxies, while another might apply to distant galaxies. State what kind of galaxy each method applies to.

(b) A galaxy has a stellar population with unreddened color $(\mathrm{B}-\mathrm{V})_0$ but a $(\mathrm{B}-\mathrm{V})_{\mathrm{obs}}$ that is redder due to internal dust. The dust is in a single thin slab such that half the stars lie in front of the slab and half lie behind. The stelar populations on both sides are identical. What is the maximum color excess $\mathrm{E}(\mathrm{B}-\mathrm{V})_{\mathrm{obs}} = (\mathrm{B}-\mathrm{V})_{\mathrm{obs}} - (\mathrm{B}-\mathrm{V})_0$ that can be observed? Assume that the intrinsic ratio of total to selective absorption ($\mathrm{R} = \mathrm{A_V}/\mathrm{E}(\mathrm{B}-\mathrm{V})$) of the dust within the slab is the standard value 3.0.

7. **297/202/PHYS213 Electromagnetism and Plasma Physics**

A neutral pion decays into two photons via $\pi^o \rightarrow 2\gamma$, where each photon has energy $m_\pi c^2/2$ in the pion's rest frame. If a pion is observed to be moving with some (possibly relativistic) velocity $c\beta$, what is the probability that one of the decay photons will be observed to have energy between ϵ and $\epsilon + d\epsilon$?

8. **297/202/PHYS213 Electromagnetism and Plasma Physics**

In its own rest frame, a dilithium molecule will emit dipole radiation if an electromagnetic field is present. In the molecule's rest frame, the power radiated is $P = k\mathbf{E} \cdot \mathbf{A}$, where k is a constant, $\mathbf{E}$ is the electric field, and $\mathbf{A}$ is the vector potential. Find an explicitly Lorentz covariant expression for the power radiated by a dilithium molecule, valid in any inertial frame of reference moving with velocity v (and 4-velocity u^μ) relative to the molecule.

9. **297/202/PHYS213 Electromagnetism and Plasma Physics**

A cubic box, of size $L(t)$, contains a fully ionized plasma, a uniform magnetic field $\mathbf{B}$, and one fast-moving electron with energy (possibly relativistic) E moving perpendicular to the field and radiating energy because of its acceleration by the field. At what rate dL/dt should the box be squeezed (while maintaining its cubical shape) in order for the fast moving electron to stay at the same energy?

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