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ASTRONOMY & ASTROPHYSICS EXAM 2009: BASIC PART

This is a 3 hour exam, with 10 questions. It is required that *all* questions be answered. Each question is identified at the top with the course number. Please use only one side of each page for your answers. If you need to extend your answer to more than one page, continue your work on one of the additional pages supplied during the exam. Be sure to put your name on every page that you turn in and, if you need to use additional pages, add both the problem number and your name at the top of each page.

You may use a hand calculator on this exam.

ASTROPHYSICS EXAM INFORMATION SHEET

Physical constants:

speed of light in vacuum	c	2.998×10^8 m/s = 2.998×10^{10} cm/s
Gravitational constant	G	6.67×10^{-11} m^3/kg s^2 = 6.67×10^{-8} cm^3/g s^2
Elementary charge	e	1.60×10^{-19} C = 4.80×10^{-10} esu
Planck constant	h	6.625×10^{-34} Js = 6.625×10^{-27} erg s
Fine structure constant	$\alpha = e^2/\hbar c$	1/137
Boltzmann constant	k	1.38×10^{-23} J/K = 1.38×10^{-16} erg/K
Gas constant	$\mathcal{R}$	= 8.32×10^7 erg K^{-1} mole^{-1}
Electron mass	m_e	9.11×10^{-31} kg = 9.11×10^{-28} gm
Proton mass	m_p	$1836 m_e$
Electron classical radius	$r_e = e^2/m_e c^2$	2.82×10^{-15} m = 2.82×10^{-13} cm
Compton wavelength	$h/m_e c$	2.426×10^{-12}m = 2.426×10^{-10}cm
Bohr radius	$a_B = \hbar^2/m_e e^2$	0.529×10^{-10} m = 0.529×10^{-8} cm
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.79×10^{-11} MeV/T
Rydberg energy	$m_e c^2 \alpha^2/2$	13.6 eV
Stephan Boltzmann const.	$\sigma_{SB} = 2\pi^5 k^4/15 c^2 h^3$	5.67×10^{-8} J/s m^2 K^4 = 5.67×10^{-5} erg/s cm^2 K^4 s
radiation constant	$a = 4\sigma_{SB}/c$	7.56×10^{-15} erg cm^{-3} K^{-4}
Thompson scattering	$\sigma_T = (8\pi/3) r_e^2$	6.65×10^{-29} m^2 = 6.65×10^{-25} cm^2
Avogadro number	N_A	6.022×10^{23} mol^{-1}

Astrophysical Quantities:

$M_\odot$	2×10^{33} g
$L_\odot$	4×10^{33} erg s^{-1}
$R_\odot$	7×10^{10} cm

Unit conversions:

electron volt	1.60×10^{-12} erg
year	3.15×10^7 s
Joule	10^7 erg
arc second	4.848×10^{-6} radians
Angstrom	10^{-8} cm
1 AU	1.50×10^{13} cm
parsec	3.08×10^{18} cm

Other useful information:

sound speed in air at 300^o K	330 m/s	3.30×10^4 c/s
atmospheric pressure	$1. \times 10^5$ N/m^2	
acceleration of gravity	9.8 m/s^2	980 cm/s^2

Equations of interest:

Maxwell's equations	$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \qquad \nabla \cdot \mathbf{H} = 0$
	$\nabla \times \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \cdot \mathbf{E} = 4\pi\rho$
ideal gas	$P = \rho kT/(\mu m_p) = \rho \mathcal{R} T/\mu$
blackbody	$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}$
blackbody radiation density	$u = (4\sigma_{SB}/c)T^4 \equiv a_B T^4$
first law	$dQ = dE + PdV$
Schrodinger's equation	$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2\mu}\nabla\Psi + U(x,y,z)\Psi$
	$(\frac{\hbar^2}{2\mu})\nabla\Psi + [E - U(x,y,z)]\Psi = 0$
Friedmann's Equation	$H^2 = H_0^2\left[\frac{\Omega_M}{a^3} + \frac{\Omega_K}{a^2} + \frac{\Omega_R}{a^4} + \Omega_\Lambda\right]$

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1. **A220A: STELLAR STRUCTURE AND EVOLUTION – J. Fortney**

In the diffusion approximation, the radiative temperature gradient is written:

$$\frac{dT}{dm} = \frac{-3}{64\pi^2 ac}\frac{\kappa L_{\mathrm{r}}}{r^4 T^3} \tag{1}$$

(a) In a post-main sequence Sun-like star, ascending the red giant brach, why does the helium core have a smaller temperature gradient than the hydrogen burning shell above it? Ignore any issue of energy loss due to neutrino emission.

(b) What particular term is important for causing outer convective zones to grow in low-mass stars? What specifically is going on?

(c) What particular term is important to causing inner convective zones to grow in high-mass stars? What specifically is going on?

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2. **A220A: STELLAR STRUCTURE AND EVOLUTION – J. Fortney**

One can derive a characteristic timescale for a process by dividing a given quantity by the time rate of change of that quantity. One important timescale for stars is the "dynamical" or "free-fall" timescale. Here our quantity of interest is the radius of a star. With that as a starting point,

(a) Derive a simple relation for the dynamical timescale. Then, for parts b) and c) just below, order the objects from the shortest to the longest dynamical time.

(b) Main Sequence Sun, Red Giant Sun, white dwarf Sun.

(c) Main sequence stars: 0.2 $M_{\odot}$, 1 $M_{\odot}$, 5 $M_{\odot}$.

(d) Another timescale is the nuclear timescale, which is basically the main sequence lifetime. Where does the approximate main sequence lifetime relation, $t_{\rm MS} \propto M^{-2.5}$, come from?

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3. **240A Galactic and Extragalactic Stellar Systems – C. Rockosi**

You measure the size, θ, the magnitude (from which you can get a calibrated flux, F) and the velocity dispersion, σ, for the elliptical and spiral galaxies in a cluster of galaxies at redshift $z = 0.8$. The sizes and magnitudes are measured so as to include a constant fraction of the light for all galaxies of a given type, be it spiral or elliptical.

(a) Write down the scaling relations appropriate for each type of galaxy that relate the luminosity, L, of galaxy to its dynamical mass. Show how they can be constructed from the above list of measured quantities to be independent of distance. What are the main assumption(s) required?

(b) You have a data for a local ($z \sim 0$) sample. Describe how these relations and be used to measure the relative change in the stellar populations of the galaxies between the local sample and the distant one: i) what is the parameter for which you can most directly measure the evolution? ii) How can you interpret your measurement of evolution (or lack thereof) in terms of the stellar populations?

(c) One limitation to your measurement is the intrinsic scatter in the scaling relations you wrote down in part a). For both the spirals and the ellipticals, i) state whether there are other relations for either type of galaxy that have smaller intrinsic scatter and ii) what that implies for the relation among the physical parameters of each type of galaxy.

4. **240A Galactic and Extragalactic Stellar Systems – C. Rockosi**

A spherical, relaxed galaxy of mass M_G, made up of equal-mass stars, has velocity dispersion σ_G and radius R_G. The density of stars in the central galaxy is low, so encounters between stars are very infrequent. This galaxy has a population of small satellite galaxies of mass M_s orbiting in its gravitational potential. These satellites are initially on orbits that are beyond R_G, with mean distance from the center of mass of the system r_{Gs}. The satellite mass M_S is negligible in comparison with M_G. You can assume that the satellites follow their orbits undisturbed until something happens to change their direction of motion and send them plunging inward, where they interact with and are eventually absorbed into the central galaxy via dynamical friction. You can assume mass follows light for this problem.

(a) What is the energy input to the central galaxy when it accretes a single satellite galaxy?

(b) How many such satellites, N, does the central galaxy need to accrete in order to raise its velocity dispersion by 5%? You can't calculate the number, but find an expression for N using the variables given.

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5. **ASTR 204B Physics of Astrophysics I – G. Laughlin**

In this question, we delve briefly into the basic structure of a *self-luminous* accretion disk whose mass is small in comparison to the mass of the object that it is orbiting.

(a) Consider a mass element in the accretion disk of a non-luminous star of mass M. Imagine that the element falls from radius $r + dr$ to r. Write down an expression for the gain in thermal energy of the mass element as a result of this inward radial shift.

(b) Assuming that the disk radiates as a thermal blackbody at the same radius where the gravitational energy is liberated, write an expression for the luminosity from the annulus in the disk.

(c) If we assume a steady accretion rate through the disk, use your results in a) and b) above to obtain an expression for the radial temperature dependence of the disk.

(d) How would you expect this temperature dependence to change if the disk is instead *inviscid*, and passively re-radiates the energy it gets from a luminous central star:

6. **ASTR 204B Physics of Astrophysics I – G. Laughlin**

If we ignore viscous and radiative forces, Newton's Second Law in a continuous fluid takes the simple form

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \Phi - \frac{1}{\rho} \nabla P \, .$$

(a) Clearly describe the physical meaning of each term in the above.

(b) Write down the definition of the vorticity, $\vec{\omega}$. A flow with $\vec{\omega} = 0$ everywhere has a very special property. What is that property?

(c) Drawing on the vector identity

$$\mathbf{u} \cdot \nabla \mathbf{u} = \nabla(\frac{1}{2}|\mathbf{u}|^2) + (\nabla \times \mathbf{u}) \times \mathbf{u} \, ,$$

and Newton's Second Law as given above, derive the condition on the equation of state that allows for a global conservation of vorticity.

(d) Give an astrophysical example of a situation where the flow would conserve vorticity to an excellent degree.

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7. **ASTR 204A Physics of Astrophysics II – G. Laughlin**

The so-called "classical radius" of an electron is given by

$$r_e = \frac{e^2}{m_e c^2}\,.$$

This "radius" provides a fiducial cross section for the interaction of photons with matter.

(a) Using a reasonable estimate for the electron scattering cross section associated with the above "radius", and assuming a completely ionized hydrogen gas, estimate the opacity in the center of the Sun. [Throughout this problem, please be sure to mention your justification when you adopt a "ball-park" (that is, rough) estimate for any particular physical quantity.]

(b) Using your estimate for the opacity, compute a mean free path for photons for representative conditions in the solar interior.

(c) If nuclear reactions in the Sun were to suddenly cease, estimate the characteristic time, τ, that would be required for us to notice a change in the Sun's luminosity.

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8. **ASTR 204A Physics of Astrophysics II – G. Laughlin**

Maxwell's Equations are written on the formula sheet for this exam.

(a) Show that Maxwell's equations support wave solutions that can propagate in a vacuum, and which have functional form $\exp^{i(\vec{k}\cdot\vec{r}-\omega t)}$.

(b) We can see with our own eyes that light is "real". In light of this, explain the physical interpretation of your complex-valued solution.

(c) Derive a dispersion relation for your wave solution.

(d) Explain the distinction between group and phase velocity. Do they differ in the case at hand?

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9. **ASTR 202 Electromagnetism and Plasma Physics – E. Ramirez-Ruiz**

This problem touches on the idea of quasi Neutrality.

(a) We asserted in class that in MHD, $n_e \approx n_i$ to high accuracy. Show from applying dimensional analysis to Gauss' Law and Faraday's Law that the fractional density imbalance can be written in the form

$$\frac{n_e - n_i}{n_e} \sim \frac{\omega_g}{\omega_p} \frac{1}{\omega_p T},$$

where $\omega_g = eB/(m_e c)$ is the electron cyclotron frequency, $\omega_p = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency, and T is the characteristic timescale in the system we are considering.

(b) As a practical application, consider solar flares, which are observed to fluctuate in brightness over 10^{-2} s. Assume $B = 100$G, $n_e = 10^8$ cm^{-3}. In developing a theory for the uctuations, would it be reasonable to assume $n_e = n_i$?

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10. **ASTR 202 Electromagnetism and Plasma Physics – E. Ramirez-Ruiz**

According to our discussions in class, cosmic rays propagate diffusively within the Galaxy. Let us apply standard diffusion theory and see what it predicts. Suppose a burst of particles injected at $t = 0$ evolves in spatial density n according to the diffusion equation

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2}$$

where D, the diffusion coefficient, can be written in terms of the particle velocity v and mean free path to scattering λ as $D = \lambda v$.

(a) Derive an expression for the diffusion time τ. Calculate λ such that $\tau = 10^7$ yr if the cosmic ray gradient scale length L is 1 kpc. Estimate how many scatterings a cosmic ray undergoes in time τ.

(b) Scattering from moving fluctuations leads to a net gain in energy (a process known as Fermi acceleration). If the flucutation velocities V are randomly oriented in direction and have mean amplitude V, the energization rate is of order

$$\frac{dE}{dt} = \frac{V^2}{\lambda c}E$$

Take $V \sim 10$ km s^{-1} and use the results of [a] to estimate the fractional change in particle energy $\Delta E/E$ during the confinement time τ. Is Fermi acceleration during propagation likely to be an important mechanism of cosmic ray acceleration? Devise an observational test of whether cosmic ray energy tends to increase with age.

(c) The rate at which an ultrarelativistic electron loses energy to synchrotron radiation in the Galactic magnetic field can be written in terms of its Lorentz factor γ as

$$\frac{d\gamma}{dt} = \frac{4}{3}\frac{\sigma_T}{mc}\gamma^2 U_B$$

where σ_T the Thomson cross section and U_B is the magnetic energy density. Estimate the electron energy at which characteristic energy loss time $E/(dE/dt)$ is the same as the confinement time. Are synchrotron losses likely to have an important effect on the Galactic synchrotron spectrum?

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